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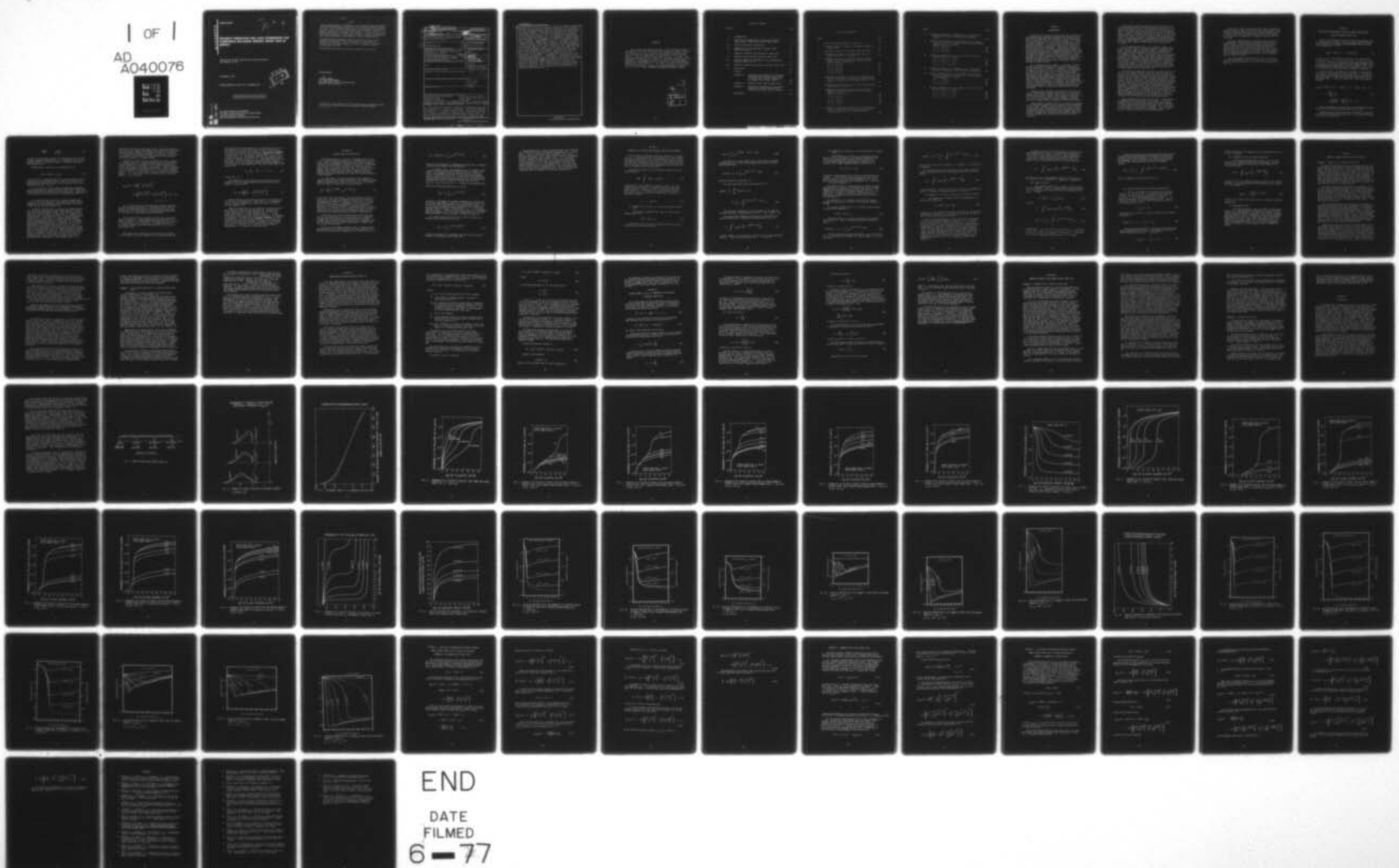

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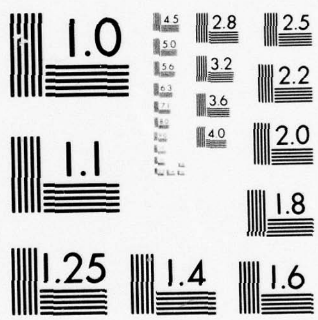

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RELIABILITY PREDICTION AND COST OPTIMIZATION FOR COMPOSITES INCLUDING PERIODIC PROOF TESTS IN SERVICE

VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
BLACKSBURG, VA 24061

NOVEMBER 15, 1976

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FOR THE DIRECTOR

S. W. Tsai

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Chief, Mechanics & Surface Interactions Branch
Nonmetallic Materials Branch

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An exploratory reliability analysis of composites under periodic proof tests in service is presented. The ultimate strength of composites is a random variable having a two-parameter Weibull distribution. A residual strength model is employed to describe the strength degradation of composites under service loads. Fatigue failure occurs as soon as the residual strength of composites is exceeded by service loads, such as gust turbulence for

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transport-type aircraft and manuever loads for fighter aircraft. Both the gust and maneuver loads are considered as random loads and their exceedance curves obtained from field data are used in the present analysis. Meanwhile, the composites are subjected to periodic proof tests in order to eliminate weak components and to ensure an acceptable level of reliability. When a component fails under the proof test, a new component is manufactured and proof-tested for replacement, so that the strength of composites is renewed. Such a renewal process is accounted for in the present analysis. Taking into account all the random variables (ultimate strength and residual strength), strength degradations, service loads, proof tests and renewal processes, the probability of structural failure in service is derived. Two numerical examples, a boron/titanium bonded joint of fighter aircraft and a glass/epoxy component of a transport-type aircraft, are worked out to demonstrate the significant influence of the proof load level and the number of periodic proof tests on the fatigue reliability of composites. Furthermore, a formulation for the optimal periodic proof test is presented. The optimal proof load level and the optimal number of periodic proof tests are obtained by minimizing the total expected (statistical average) cost, while the constraint on the allowable level of structural reliability is satisfied. It is demonstrated by numerical examples that significant cost savings and reliability improvement for composite structures can be achieved by the application of the optimal periodic proof test.

FOREWORD

This report represents one part of the research results sponsored by the Air Force Materials Laboratory under contract No. F33615-75-C-5112 to Virginia Polytechnic Institute and State University, Department of Engineering Science and Mechanics. The research was performed by Dr. J. N. Yang. The final preparation of the report was conducted during the period September 1 to November 15, 1976, at The George Washington University, Department of Civil, Mechanical and Environmental Engineering with which Dr. J. N. Yang is presently associated. The project engineer for the Air Force Materials Laboratory is Dr. S. W. Tsai, Chief, **Mechanics and Surface Interaction Branch.**

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INTRODUCTION

Unlike metallic aircraft structures where NDI techniques for crack detection and maintenance procedures for fracture control have been well developed, the technique of fatigue damage detection for composites is still in a primitive stage. As a result, it has been advocated [1-3] that proof testing may be a very promising approach for the safety control and certification of composite structures, in particular, when critical locations of the structure are not inspectable. The purpose of performing periodic proof tests is to eliminate weak components whose residual strength has progressively degraded under service loads, thus ensuring an acceptable level of composite reliability in service. The theory of single proof testing and optimization have been developed for metallic materials, particularly for application to pressure vessel design [e.g., 4-8]. The fatigue reliability analysis for composites under periodic proof tests, however, has not been available to date.

It is the purpose of this paper to (i) present a fatigue reliability analysis methodology for composites subjected to periodic proof tests in service, in particular, the improvement of composite reliability in service resulting from the application of period proof tests, and (ii) establish an optimum periodic proof test, based on the cost and reliability criteria. The present analysis enables one to determine the optimal proof load level and the optimal number of periodic proof tests such that the total expected cost is minimized, while an acceptable level of composite reliability is satisfied. The present effort is relevant to the establishment of rational design criteria, cost and risk minimization, as well as maintenance procedures and certification plans for composites in application to aircraft structures.

Fatigue has been a problem in the design of aircraft structures. This problem is further complicated by the statistical (random) nature of most loading inputs to aircraft structures as well as the statistical nature of the strength of composites. As a result, the probabilistic approach becomes an important tool for a rational analysis and design of composite structures.

Fatigue damage in composites may be revealed by the degradation of residual strength [1,10-14]. However, the determination of the residual strength, nondestructively, may not be feasible in practice [13-14]. Therefore, proof testing becomes a viable and promising method for eliminating weak components when their residual strengths have fallen below the allowable safety level and thus increasing the reliability and safety of composite structures.

The ultimate strength of composites has been shown to follow a two-parameter Weibull distribution reasonably well, and it is used in the present analysis. The parameters associated with the Weibull distribution are determined from the results of laboratory tests. A residual strength degradation model [1,9] has been employed herein to describe the strength degradation of composites under service loads. The analysis, however, can incorporate any model of strength degradation. The parameters associated with the residual strength model are determined from the results of laboratory tests under flight-by-flight random service load spectra [1,10-12].

For transport-type aircraft, such as bombers, tankers, etc., failure may occur when the residual strength is exceeded by gust loads. The gust loads are considered herein as stationary composite Gaussian random processes as discussed in Refs. 15-21. For fighter aircraft, failure may occur when the residual strength is exceeded by maneuver loads. The maneuver loads are approximated by a finite number of Gaussian random processes with different intensities (standard deviations) and durations as modeled by Press [21-22]. The failure mode discussed herein is referred to as the first-passage or first-excursion failure in random vibration [23-24]. Hence, as the service time increases, the residual strength decreases progressively due to fatigue damage and thus increasing the failure rate with time [19].

Proof testing is performed at periodic intervals [see Fig. 1], and only those components that survive the proof test are used in service. As a result, after each proof test the statistical distribution of the residual strength, including the ultimate strength prior to service, of the surviving components changes and is truncated at the proof load level. Such a truncation effect eliminates the lower tail of the distribution function (or density function) up to the proof load level and thus increases the reliability of components. The composite may, however, be damaged during each proof testing depending on the proof load level. A possible damage model is suggested and is incorporated in the present analysis. Consequently, the statistical distribution of the residual strength not only changes with time due to the accumulation of fatigue damage (strength degradation) but also changes after each proof testing due to the truncation and possible damaging effect.

When a component fails under the proof test, a new component is manufactured and proof-tested for replacement. If the new component fails also under proof testing, then another component is further manufactured and proof-tested again until a new article which survives proof testing is obtained. Thus, the strength of the component is renewed after replacement. Such a renewal process is taken into account in the present analysis.

Taking into account the random variables (static strength and residual strength), random service loads (gust and maneuver loads), strength truncations due to proof tests, strength degradations due to service loads, and the renewal processes due to replacements, the solution for the probability of composite failure in service is derived.

Finally, a formulation for the optimal periodic proof test is established. The optimal proof load level and the optimal number of periodic proof tests are obtained by minimizing an objective function, which is the total expected (statistical average) cost, while the constraint on allowable level of composite reliability in service is satisfied. The total expected cost consists of the expected cost of proof tests, the expected cost of components destroyed by proof tests, and the expected cost of failure in service.

The experimental verifications for the residual strength degradation model and the theory of periodic proof tests discussed in this report are presented in Ref. 28.

SECTION II

STATISTICAL DISTRIBUTION OF RESIDUAL STRENGTH UNDER SERVICE LOADS AND PERIODIC PROOF TESTS

Let R_0 be the ultimate strength (or stress) of composites prior to service and proof testing. It has been shown that the statistical distribution of R_0 follows the two-parameter Weibull distribution reasonably well [1-2],

$$F_{R_0}(x) = P[R_0 \leq x] = 1 - \exp[-(x/\beta)^\alpha] \tag{1}$$

in which $F_{R_0}(x)$ is the distribution function of R_0 indicating the probability that R_0 is smaller than or equal to an arbitrary value x. In Eq. 1, α and β are, respectively, the shape parameter and the scale parameter (or characteristic ultimate strength). Both α and β are determined from experimental data. A catalog of α and β values for various composite laminates has been compiled in Reference 1.

Prior to service, the composite is subjected to proof testing referred to as the initial (or first) proof test. Let R(0) be the strength of components that survive the initial proof test at a proof load level, r_0. Then, the distribution function of R(0) can be obtained from that of R_0 given by Eq. 1 by a truncation as follows;

$$\begin{aligned} F_{R(0)}(x) &= P[R(0) \leq x] = 1 - P[R(0) > x] = 1 - P[R_0 > x | R_0 > r_0] \\ &= 1 - \frac{P[R_0 > x]}{P[R_0 > r_0]} \\ &= 1 - \exp\left\{\left(\frac{r_0}{\beta}\right)^\alpha - \left(\frac{x}{\beta}\right)^\alpha\right\} \; ; \text{ for } x \geq r_0 \end{aligned} \tag{2}$$

After the component is put into service, the strength decreases progressively due to fatigue damage under service loads.

Assuming that the slope of the residual strength of composites is inversely proportional to some power c-1 of the residual strength itself, one can write [3,28]

$$\frac{dR(t)}{dt} = - \frac{A}{R^{c-1}(t)} \tag{3}$$

in which t is the number of flights (or flight hours), R(t) is the residual strength of composites at t, c is believed to be a material constant, and A is a function of material properties and service load characteristics.

Integrating Eq. 3 from t_0 to t, one obtains for $c \neq 1$,

$$R^c(t) = R^c(t_0) - \Phi(t-t_0) \tag{4}$$

in which $\Phi = cA$ is a complicated function of both the material properties and service load characteristics. Both c and Φ have to be determined from experimental results under flight-by-flight service loading spectra [e.g., 1,10-12]

Eq. 4 is similar to the residual strength model proposed by Halpin and Waddoups for composite materials [9,1,10-12], except that it was derived on the assumption of crack propagation. Experimental verifications for such a model is presented in Ref. 28.

It is emphasized that a particular residual strength model, such as Eq. 4, is not essential to the present development of the reliability analysis methodology, since other models of residual strength can be similarly accounted for.

It should be noticed that Φ is a function of material properties and service loads, such as ground loads, ground-air-ground loads, gust loads and maneuver loads. Since most of the service load input involves statistical variability, Φ is theoretically a random variable. It follows then from Eq. 4 that associated with a sample value x of R(0) there is no one-to-one corresponding sample value y of R(t) since Φ is a random variable, and hence the strength degradation follows a random path. It is mentioned, however, that the assumption that Φ is deterministic, is a good approximation if the statistical dispersion of Φ is small or negligible (a deterministic constant has zero statistical dispersion). It has been shown in Ref. 25 that the statistical dispersion (coefficient of variation) of Φ indeed diminishes as $O(1/\sqrt{n})$ where n is the number of load cycles. Since for practical applications, the number of load cycles within the service life is extremely large, it may be reasonable to neglect the effect of the statistical dispersion of Φ. The assumption that Φ is a

deterministic constant has been used in Refs. 1,10-12 for the analysis of test data. Recent investigations also indicate [3] that such an assumption may be reasonable. Although the statistical dispersion of Φ is neglected in the present analysis for the sake of simplicity in mathematical presentation, the present analysis can account for the situation where Φ is a random variable.

Supposing that proof testing is performed at periodic intervals of T flights, see Fig. 1, the distribution function of the residual strength changes with time because of the strength degradation in service and because of the effect of proof tests. A detailed derivation for the change of the strength distribution is given in Appendix I and is schematically displayed in Fig. 2. The probability density function, $f_{R(jT)}(x)$, of the residual strength, $R(jT)$, of the component having survived up to the j+1th proof test is derived in Appendix I; with the result

$$f_{R(jT)}(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{c-1}\left(\frac{x^c + \Phi jT}{\beta^c}\right)^{\frac{\alpha}{c}-1} \exp\left\{\left(\frac{r_0{}^c + \Phi jT}{\beta^c}\right)^{\alpha/c} - \left(\frac{x^c + \Phi jT}{\beta^c}\right)^{\alpha/c}\right\} ;\; x \geq r_0 \quad (5)$$

$$j = 0, 1, 2, \ldots$$

Eq. 5 is derived under the condition that the proof load level, r_0, does not damage the composite. Experimental data [1-3,26-27] indicate that the damage resulting from proof testing may be neglected if the proof load level r_0, is below 90% of the characteristic strength for $\pi/4$ glass/epoxy and other laminates.

As the proof load level increases, the proof test itself will damage the composite. A damage model under the proof test is presented in Appendix II and the probability density function $f_{R(jT)}(x)$ is derived in Eq. II-5. Note that Eq. II-5 degenerates into Eq. 5 when no damage occurs under the proof test, i.e., $D_1=1$, $D_2=1$ [see Appendix III].

Other quantities of importance in the reliability assessment are the probability that the component will be destroyed under each

proof test as well as the probability of surviving each proof. Let the components, which pass the initial (or the first) proof test and are put into service, be referred to as the original components. Let B_j^* be the probability that an original component will survive j+1 proof tests, i.e., the original component will survive up to jT flights [see Fig. 1]. Furthermore, let B_j be the probability that an original component will fail (or be destroyed) during the j+1th proof test. Since the probability of being destroyed during the j+1th proof test is the difference between the probability of surviving j+1 proof tests and the probability of surviving j proof tests, it is obvious that

$$B_j = B_{j-1}^* - B_j^* \quad ; \quad j = 1, 2, \cdots \tag{6}$$

in which $B_0^* = 1.0$.

The probability of surviving up to the j+1th proof test performed at jT for an original component has been derived in the Appendix I; with the result

$$B_j^* = \exp\left\{\left(\frac{r_0}{\beta}\right)^{\alpha} - \left(\frac{r_0^{\,c} + \Phi jT}{\beta^c}\right)^{\alpha/c}\right\} \tag{7}$$

When the damage occurs during each proof test, B* has been derived in Eq. II-6 of Appendix II. Note that Eq. II-6 degenerates into Eq. 7 when no damage occurs under proof tests, i.e., D_1=1, D_2=1 [see Appendix III].

Under the renewal policy, a new component is manufactured and proof-tested to replace the one destroyed by the proof test. The new component may fail under the proof test, in which case another new component is further manufactured and proof-tested until a new article that survives is obtained. The component for replacement is referred to as the renewal component in order to differentiate it from the original component. As a result, the strength of the structural component is renewed after replacement. Such a renewal process must be taken into account. Under the renewal policy, B_j derived in Eq. 6 represents the percentage of replacement at jT flights (or j+1th proof test).

SECTION III

SERVICE LOADS AND FAILURE RATE

As mentioned previously, failure of components occurs as soon as the residual strength of composites is exceeded by the service loads. Hence, the exceedance characteristics of service loads are essential to the assessment of composite reliability. For transport-type aircraft, the critical load which may exceed the residual strength level is essentially the gust load. For fighter aircraft, however, the exceedance of the residual strength may be due to maneuver loads. Since both gust loads and maneuver loads have different statistical characteristics, they are discussed separately in the following;

(1) Gust Loads: The gust turbulence has been modeled as a composite Gaussian random process [15,17-21]. The expected (average) number of upcrossings of a load (or stress) level, x, by the gust load per flight (or flight hour) can be expressed as

$$F_s^*(x) = N^*\left[u_1 e^{-(x-x_0)/\sigma_1} + u_2 e^{-(x-x_0)/\sigma_2}\right] \tag{8}$$

in which N^* is the total number of gust load cycles per flight (or flight hour), and u_1 and u_2 represent the fractions of clear air turbulence and thunderstorm turbulence, respectively, with associated intensities σ_1 and σ_2. Parameters u_1, u_2, σ_1 and σ_2 are referred to as turbulence field parameters and are specified in Reference 15 for various altitudes. x_0 appearing in Eq. 8 is the mean stress (or load) due to one g loading. The graphical representation of Eq. 8 is referred to as the exceedance curve, which is established from measured turbulence data [e.g., Reference 16].

In practice, the residual strength level is much higher compared to the intensities of σ_1 and σ_2 and hence it is reasonable to assume that the crossings (or excursions) of the residual strength level by the gust load are statistically independent [19]. This is referred to as the Poisson approximation in random vibration and will be used herein.

Let $R(\tau)$ be the residual strength at the τth flight, then the expected number of upcrossings over the residual strength at the τth flight by the gust load, denoted by $h(\tau)$, follows from Eq. 8 as

$$h(\tau) = F_s^*[R(\tau)] = N^* \sum_{i=1}^{2} u_i e^{-[R(\tau)-x_0]/\sigma_i} \qquad (9)$$

Because of the assumption of independent crossings, $h(\tau)$ is referred to as the failure rate or the risk function.

(2) Maneuver Loads: The exceedance curves for maneuver loads depend on the mission operational characteristics of fighter aircraft. The exceedance curves are obtained from the statistical analysis of measured data [22,15]. A typical maneuver exceedance curve is given in Fig. 3 for F-111 fighters [22]. According to the method of Press [10,21], the exceedance curve can be approximated by the summation of a finite number of straight lines in a log $F_s^*(x)$ vs. $(x-x_0)^2$ plot (see Fig. 3). Therefore the exceedance curve $F_s^*(x)$ can be expressed analytically as follows;

$$F_s^*(x) = N^* \sum_{i=1}^{m} u_i e^{-(x-x_0)^2/2\sigma_i^2} \qquad (10)$$

in which m is the number of straight lines depending on the accuracy required for approximation, and u_i and σ_i are measured directly from the straight lines [see Reference 21 for detailed discussion]. Eq. 10 implies that the maneuver load is decomposed into m segments of Gaussian random processes with intensities $\sigma_1, \sigma_2, \cdots, \sigma_m$ and durations $N^*u_1, N^*u_2, \cdots, N^*u_m$, since each term in Eq. 10 is a stationary Gaussian exceedance curve. This type of approximation will be used herein expediently for the assessment of composite reliability.

As a result, the failure rate, $h(\tau)$, at the τth flight for a fighter aircraft follows from Eq. 10 as

$$h(\tau) = N^* \sum_{i=1}^{m} e^{-[R(\tau)-x_0]^2/2\sigma_i^2} \qquad (11)$$

in which the assumption of independent crossings over the residual strength by the maneuver load has been made.

Considerable amount of data on fighter maneuver loads indicates that unlike the gust load, the maneuver exceedance curve is not symmetric. This is an indication that the maneuver load is not a Gaussian random process, and further investigation for the statistical model of maneuver loads is needed [19]. For the reliability analysis, however, the approximation of the positive exceedance curve by the method of Press [21], as discussed previously, seems to be acceptable as a first order approximation. Note that the negative exceedance curve is disregarded, since it is usually much smaller than the positive exceedance curve. The advantage of such an approximation [18] discussed previously lies on its mathematical simplicity, which is essential for practical implementation.

SECTION IV

PROBABILITY OF FAILURE UNDER PERIODIC PROOF TESTS IN SERVICE

Proof testing is performed at periodic intervals of T flights, see Fig. 1. The probability of failure within each service interval will be computed first and then the probability of failure will be evaluated as a function of service time. In what follows, the failure rate of Eq. 9 (transport-type aircraft) will be used for simplicity in derivation and the results associated with the failure rate of Eq. 11 (fighters) will be given later.

The following formula for the conditional probability will be used frequently,

$$P[A] = \int_{-\infty}^{\infty} P[A|X = x]f_X(x)dx \tag{12}$$

in which P[A] is the probability of failure. $P[A|X = x]$ is the probability of failure given (or under the condition that) the random variable X is equal to a value x, and $f_X(x)$ is the probability density of X. Furthermore, if the total failure rate in one service interval is H, then the probability of failure P_f in that service interval is

$$P_f = 1 - \exp(-H) \tag{13}$$

(1) Probability of Failure, p_1, in the First Service Interval (0,T)

The residual strength $R(\tau)$ at the τth flight in (0,T), follows from Eq. 4 as

$$R^c(\tau) = R^c(0) - \Phi\tau \tag{14}$$

Substitution of Eq. 14 into Eq. 9 yields the failure rate at τ within the 1st service interval (0,T)

$$h(\tau) = N^* \sum_{i=1}^{2} u_i e^{-\{[R^c(0) - \Phi\tau]^{1/c} - x_0\}/\sigma_i} \qquad (14b)$$

Since R(0) is a random variable with a statistical distribution function given by Eq. 2, Eq. 14b can formally be written as a conditional failure rate,

$$h[\tau|R(0) = x] = N^* \sum_{i=1}^{2} u_i e^{-[(x^c-\Phi\tau)^{1/c} - x_0]/\sigma_i} \qquad (15)$$

where the condition is that R(0) = x.

The total conditional failure rate within (0,T) is,

$$H_1[R(0) = x] = \int_0^T h[\tau|R(0) = x]\, d\tau$$

$$= N^* \sum_{i=1}^{2} u_i \int_0^T e^{-[(x^c-\Phi\tau)^{1/c} - x_0]/\sigma_i}\, d\tau \qquad (16)$$

The conditional probability of failure within (0, T), under the condition that R(0) = x, follows from Eq. 13 as $1 - \exp\{-H_1[R(0) = x]\}$.

The unconditional probability of failure, p_1, within the first service interval (0,T) is obtained by the application of Eq. 12,

$$p_1 = \int_{r_0}^{\infty} f_{R(0)}(x) \left[1 - e^{-H_1[R(0) = x]}\right] dx \qquad (17)$$

in which $H_1[R(0) = x]$ is given by Eq. 16 and $f_{R(0)}(x)$ is the probability density function of R(0) given by Eq. 5 with j=0.

(2) Probability of Failure, p_2, in the second service interval (T,2T)

The probability of failure, p_2, in the second service interval (T,2T) is attributed to two different populations; (i) the original component that survives the 2nd proof test at T, (ii) the renewal component manufactured at T because of the failure of the original component under the proof test at T,

$$p_2 = B^*_1 V_2 + B_1 p_1 \tag{18}$$

in which V_2 is the probability of failure in the second service interval (T,2T) for the original component that survives the 2nd proof test at T with the surviving probability B^*_1. Hence, $B^*_1 V_2$ is the contribution of failure probability in (T,2T) from the original component.

The second term $B_1 p_1$ is contributed by the renewal component manufactured at T, where B_1 is the probability that the original component will be destroyed under the 2nd proof test at T. The probability of failure of the renewal component in (T,2T) is p_1 given by Eq. 17, which is referred to as the renewal failure probability. Both B^*_1 and B_1 have been derived in Eqs. 6 and 7. Note that $B^*_1 + B_1 = 1.0$.

To evaluate V_2, it is necessary to estimate the residual strength and failure rate in (T,2T) for the original component as follows;

The residual strength $R(T+\tau)$ at $T+\tau$ flights can be expressed in term of R(T) through Eq. 1,

$$R^c(T+\tau) = R^c(T) - \Phi\tau \tag{19}$$

The failure rate at $T+\tau$ flights is obtained by substituting Eq. 19 into Eq. 9. Then, the conditional failure rate can be shown as

$$h[T+\tau | R(T) = x] = N^* \sum_{i=1}^{2} u_i e^{-[(x^c - \Phi\tau)^{1/c} - x_0]/\sigma_i} \tag{20}$$

The total conditional failure rate $H_2[R(T) = x]$ in (T,2T) given $R(T) = x$ is obtained by integrating Eq. 20 with respect to τ; with the result,

$$H_2[R(T) = x] = N^* \sum_{i=1}^{2} u_i \int_0^T e^{-[(x^c - \phi\tau)^{1/c} - x_0]/\sigma_i} d\tau \qquad (21)$$

Note that Eqs. 16 and 21 are identical, i.e., $H_2[R(T) = x] = H_1[R(0) = x]$, as expected.

The unconditional failure probability, V_2, for the original component, is obtained from Eq. 21 by the application of Eqs. 12-13 as follows;

$$V_2 = \int_{r_0}^{\infty} f_{R(T)}(x) \left[1 - e^{-H_2[R(T) = x]}\right] dx \qquad (22)$$

in which $H_2[R(T)=x]$ is given by Eq. 21 and $f_{R(T)}(x)$ is the probability density function of R(T) given by Eq. 5 with j=1.

(3) Probability of Failure in the 3rd Service Interval (2T,3T)

The probability of failure in (2T,3T), denoted by p_3, is attributed to three parts;

$$p_3 = B_2^* V_3 + B_1 p_2 + B_2 p_1 \qquad (23)$$

in which V_3 is the probability of failure in (2T,3T) for the original component that survives all of the previous proof tests, and B_2^*, B_1, B_2, p_1 and p_2 have been given previously.

Eq. 23 is self-explanatory. The first term $B_2^* V_3$ is the contribution from the original component, where B_2^* is the probability of surviving all previous proof tests up to 2T (Eq. 6). The second term $B_1 p_2$ is the contribution from the renewal component manufactured at T, where the renewal probability is B_1, and the renewal failure probability in (2T,3T) for the renewal component is p_2. The third term $B_2 p_1$ represents the contribution from the renewal component manufactured at 2T to replace the original component that fails under the 3rd proof test performed at 2T. The probability of such a replacement is B_2 and the renewal failure probability in (2T,3T) is p_1. It is important to notice that $B_2^* + B_1 + B_2 = 1.0$.

V_3 appearing in Eq. 23 can be evaluated by first expressing the residual strength, $R(2T+\tau)$, of the original component in terms of $R(2T)$, i.e., $R^c(2T+\tau) = R^c(2T)-\Phi\tau$ and by substituting it into Eq. 9 to obtain the failure rate and the conditional failure rate. Then, with the application of Eq. 12-13 in a manner similar to the derivation for V_2, it can be shown that

$$V_3 = \int_{r_0}^{\infty} f_{R(2T)}(x) \left[1 - e^{-H_3[R(2T) = x]}\right] dx \tag{24}$$

in which $f_{R(2T)}(x)$ is the probability density of $R(2T)$ given by Eq. 5 with $j=2$, and $H_3[R(2T)=x] = H_2[R(T)=x] = H_1[R(0)=x]$.

(4) General Solution

Let p_j be the probability of failure in the jth service interval $[(j-1)T, jT]$. Then, in a manner similar to the derivation for p_1, p_2 and p_3, one can derive the general recurrence solution p_j as follows;

$$p_j = B^*_{j-1} V_j + \delta_{j-1} \sum_{k=1}^{j-1} B_k p_{j-k} \tag{25}$$

in which

$$V_j = \int_{r_0}^{\infty} f_{R[(j-1)T]}(x) \left[1 - e^{-H(x)}\right] dx \tag{26}$$

$$H(x) = N^* \sum_{i=1}^{2} u_i \int_0^T e^{-[(x^c-\Phi\tau)^{1/c}-x_0]/\sigma_i} d\tau \tag{27}$$

in which $B^*_0=1$, $\delta_{j-1}=1$ for $j-1>0$ and $\delta_{j-1}=0$ for $j \leq 0$. B_j and B^*_j are given by Eqs. 6 and 7, and $f_{R(jT)}(x)$ is given by Eq. 5. Note that Eq. 27 can be integrated analytically when c is an integer.

It should be mentioned that the solutions given above are for the transport-type aircraft where the exceedance of the residual strength is due to gust turbulence. For fighter aircraft where the exceedance is due to maneuver loads, the solutions given by Eq. 25-27 still hold except that H(x) should be changed as follows;

$$H(x) = N^{*} \sum_{i=1}^{m} u_i \int_0^T e^{-[(x^c - \Phi\tau)^{1/c} - x_0]^2/2\sigma_i^2} d\tau \qquad (28)$$

which is obtained by the application of Eq. 11.

(5) Cumulative Probability of Failure and Reliability

Thus far, the probability of failure in each service interval, i.e., p_j for $j=1,2,\cdots$, has been derived. The probability of survival in each service interval is therefore $1-p_j$ for $j=1,2,\cdots$. Let r_j be the probability of survival (called reliability) in j service intervals (0,jT). Then the survival in j service intervals (0,jT) implies the survival in all intervals up to jT, i.e.,

$$r_j = \prod_{i=1}^{j} (1 - p_i) \qquad (29)$$

The probability of failure in j service intervals (0,jT), denoted by $p_f(r_0,j)$, is

$$p_f(r_0,j) = 1 - r_j = 1 - \prod_{i=1}^{j} (1 - p_i) \qquad (30)$$

Supposing that the composite is subjected to N periodic proof tests in the design service life T_0, the probability of failure within the design service life, $p_f(r_0,N)$, is

$$p_f(r_0,N) = 1 - \prod_{i=1}^{N} (1 - p_i) \qquad (31)$$

and the reliability of the composite within the design service life is $r_N = 1 - P_f(r_0,N)$.

(6) Probability of Failure Without Proof Test

Let P(t) be the probability of failure in the service time (0,t) for components without being proof-tested. The solution for P(t) can easily be derived in a similar manner as Eq. 17; with the result,

$$P(t) = \int_0^{\infty} f_{R_0}(x) \left[1 - e^{-H(R_0=x,t)}\right] dx \tag{32}$$

in which $f_{R_0}(x)$ is the probability density function of the ultimate strength R_0 and is obtained by differentiating $F_{R_0}(x)$ given by Eq. 1 with respect to x,

$$f_{R_0}(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(x/\beta)^{\alpha}} \tag{33}$$

and H[R =x,t] is equal to H(x) given by Eq. 27 in which T should be replaced by t.

(7) Non-Renewal Policy

The solutions derived in Eqs. 25-27 hold for the renewal policy, i.e., a new component is manufactured and proof-tested to replace every component which fails under proof testing. A special case of interest is that no replacement is made for the components destroyed by proof testing. In other words, the airplane becomes unavailable for service. Under such a non-renewal policy, the general solutions given by Eqs. 25-27 still hold except that the second term (summation terms) of Eq. 25 should be disregarded, since it represents the failure probability due to renewal components.

SECTION V

NUMERICAL EXAMPLES FOR RELIABILITY PREDICTION

Example 1: Bonded Joint of Fighter Aircraft Wing

A boron/titanium bonded joint has been tested under the F-111 fighter maneuver loading spectra of Fig. 3 [10,22]. The mean load (one g load) to the wing is 2.1×10^6 in-# of bending moment which results in 6.734 kips of force to the bonded joint (i.e., 10^6 in-# of bending moment produces 3.25 kips of force to the joint). The exceedance curve of maneuver loads for one flight is displayed in Fig. 3 in which the abscissa represents the square of bending moment measured from the mean load, $(\Delta BM)^2$. The ordinate represents the average number of crossings of ΔBM level per flight. It is noticed that the wing is subjected to 328 cycles of significant loads and one flight is of 3 hours duration.

According to the method of Press [21], the exceedance curve of Fig. 3 can be reasonably expressed as the superposition of two straight (dashed) lines as indicated. As a result, the exceedance curve of the maneuver load per flight can be written analytically in the form of Eq. 10 in which m=2, and u_1, u_2, σ_1 and σ_2 are measured directly from the dashed lines of Fig. 3. After appropriate conversion into forces applied to the bonded joint, it can be shown that $N^*u_1 = 15.8$, $\sigma_1 = 16.194$ kips, $N^*u_2 = 158$, $\sigma_2 = 7.89$ kips, and $x_0 = 6.734$ kips.

Test results given by Ref. 10 indicate that the ultimate strength of the bonded joint can be represented by a two-parameter Weibull distribution with the shape parameter $\alpha = 11.62$ and the characteristic ultimate strength (scale parameter) $\beta = 109$ kips. Test results on the residual strength under flight-by-flight random service spectra indicate that the form of Eq. 4 for the strength degradation is satisfactory and that c = 10.12. With appropriate transformation of the results given by Reference 10, it can be shown that $\Phi = 1.55 \times 10^7$ kips^c per flight, where R(t) and R(0) are expressed in terms of kips and t represents the number of flights. Note that one life time is equal to 1334 flights.

Supposing that the design service life of the bonded joint is 1500 flights, the probability of failure (1-reliability) of the bonded joint within the service time of t flights is computed from Eq. 32 and is plotted in Fig. 4 as a dashed curve where no proof testing is performed. Also plotted in Fig. 4 are the solid curves representing the probabilities of failure of the bonded joint when it is proof-tested only once prior to service at various proof load level, r_0. It is observed from Fig. 4 that a single, initial proof test at the proof load level of $r_0 = 100$ ksi does result in

a significant reliability improvement within the service time of 1000 flights, but it has little advantage within the design service life of 1500 flights. Therefore, periodic proof testing in service is needed in order to maintain a reasonable level of reliability within the design service life.

The probability of failure of the bonded joint under periodic proof tests in service is calculated from Eqs. 25 and 30 and plotted in Fig. 5 for various number, N, of proof tests and various proof load levels. In Fig. 5, the number of proof tests N=15 implies that proof testing is performed at every 100 flights of service, i.e., one service interval is 100 flights. Fig. 5 clearly demonstrates that significant improvement on the reliability of bonded joints can be achieved by the application of periodic proof tests and that the reliability increases as the number of periodic proof tests increases or as the proof load level increases.

Damage to the bonded joint by proof tests has been neglected in the numerical example because of lack of quantitative information regarding the damaging functions D_1 and D_2 (Appendix II). It should be accounted for later when the information becomes available.

The probability of failure, $P_f(r_0,N)$ (Eq. 31) within the design service life of 1500 flights, is summarized in Fig. 6 for various proof load levels. One significant observation is that associated with each proof load level, there is a limit on the number, N, of proof tests beyond which the reliability improvement is negligible. Such a limit sets a bound for the reliability improvement as can be observed from the asymptote of each curve in Fig. 6. Thus, once a proof load level is selected, there is an ultimate reliability improvement beyond which no improvement can be obtained. For instance, a reliability of 0.99 ($P_f(r_0,N) = 10^{-2}$) for the bonded joint can be achieved by either 2 proof tests at 100 kips or 3 proof tests at 95 kips or 5 proof tests at 90 kips. However, if the reliability of the bonded joint is specified to be greater than 0.999 ($P_f(r_0,N) = 10^{-3}$), a proof load level below 95 kips can not achieve such a reliability requirement regardless of the number of proof tests.

It is important to note that a proof load level of 80 kips for the bonded joint, which is 73.4% of the characteristic ultimate load (β = 109 kips) results in insignificant reliability improvement unless the number of proof tests is high, e.g., N=20 (see Fig. 6). According to design practice, the limit load is 66.7% of the ultimate load (the ratio is 1 to 1.5). Therefore, it follows from these observations that, for the particular example considered herein,

periodic proof testing at the limit load does not serve the purpose of reliability improvement unless the fleet size is very large or to eliminate design errors. This observation is consistent with the conclusions for metallic materials [7,8] that a low proof load level does not produce meaningful reliability improvement.

Example 2: A Glass/Epoxy Component for a Transport-Type Aircraft Wing

A glass/epoxy $\pi/4$ laminate subjected to transport-type aircraft loading spectra is considered. The ultimate strength, R_0, of such a laminate has been shown experimentally to follow a two-parameter Weibull distribution [1] with the shape parameter $\alpha = 12.3$ and the scale parameter (characteristic ultimate stress) $\beta = 53$ ksi. Test results [1] indicate that the parameter c associated with the strength degradation model (Eq. 4) is $c = 9.28$. As mentioned previously, the parameter Φ (Eq. 3) depends on the characteristics of service loads and it has to be determined by test results under flight-by-flight service load spectra. The following turbulence parameters associated with a typical gust load are used [19]; $u_1 = 0.995$, $u_2 = 0.005$, $\sigma_1 = 0.65$ ksi, $\sigma_2 = 1.8$ ksi (see Eq. 8). The mean stress (stress due to one g load) is $x_0 = 8$ ksi (Eq. 8). With the loading spectra given above as well as the test results given by Ref. 1, it is assumed that $\Phi = 40 \times 10^{11}$ ksi^c per flight hour. Note that in this example both the residual strength, R(t), and the ultimate strength, R_0, appearing in Eq. 4 have units of ksi, and t represents the number of flight hours. The structure is subjected to 600 cycles of gust loads per flight hour, i.e., $N^* = 600$ (Eq. 8).

Supposing that the design service life for such a component is 10,000 flight hours, the probability of failure within t flight hours is plotted in Fig. 7 as a dashed curve for the component without being proof-tested (Eq. 30). Also plotted in Fig. 7 as solid curves are the probabilities of failure for the components which are proof tested only once prior to service. It is observed that a single proof test at a level of 45 ksi results in a significant reliability improvement within 5,000 flight hours. However, within the design service life of 10,000 flight hours, a single proof test has little benefit and hence periodic proof test in service is necessary to maintain a reasonable level of reliability.

The probability of failure of the component under periodic proof tests in service is plotted in Fig. 8 for various proof load levels. Fig. 8 clearly demonstrates the significant improvement of composite reliability by the use of periodic proof tests. Furthermore, the composite reliability increases as the proof load level increases or as the number of periodic proof tests increases.

It should be mentioned that limited amount of test data for a $\pi/4$ glass/epoxy laminate given in Ref. 2 indicates that the damage D_1 (see Appendix II) is negligible for a proof load level r_0 = 46.75 ksi, i.e., 88% of the characteristic strength. Another set of test data for the same composite given in Ref. 1 indicates that the damage D_2 (see Appendix II) is negligible for a proof load level r_0 = 48 ksi, i.e., 90% of the characteristic strength β. In view of these results, we restrict our investigation to the case where the proof load level r_0 is below 45 ksi, so that damage to the composite resulting from periodic proof tests can be neglected.

The probability of failure, $p_f(r_0,N)$, within the design service life, i.e., 10,000 flight hours, is displayed in Fig. 9. Similar conclusions as in the case of the bonded joint of fighter have been observed; (i) associated with each proof load level, there is a limit on the number of periodic proof tests beyond which the benefit of reliability improvement vanishes, (ii) Each curve in Fig. 8 has an asymptote which is a limiting reliability one can achieve for a particular level of proof load, regardless of the number of proof tests, (iii) A proof load level at the limit load, i.e., 0.67β = 35.5 ksi, does not produce significant reliability improvement for composites, unless the fleet size is extremely large.

SECTION VI

FORMULATION FOR OPTIMAL PERIODIC PROOF TEST

It has been demonstrated in the previous section that the reliability of composites increases as the number of periodic proof tests increases or as the proof load level increases, provided that the proof load level is below the critical load level which may damage the composites. However, as the number of periodic proof tests increases, the cost of proof tests increases (including the total cost of performing proof tests, down time, nonavailability for service, etc.). Furthermore, as the level of proof load increases, the probability of destroying components under periodic proof tests increases, i.e., the expected number of components to be destroyed by periodic proof tests increases, and thus increasing the cost of replacement. Since the damage due to proof tests has not been quantitatively investigated, we shall restrict ourselves to the case where the proof load level is below the critical load level so that proof testing does not damage the composites.

Although high level of composite reliability can be achieved through the application of periodic proof tests, the costs of proof tests and replacements may be significant and have to be minimized. There is indeed a trade-off potential between the reliability of composites and the costs of proof tests and replacement. It is the purpose of this section to perform an exploratory study to establish the optimal proof load level and the optimal number of periodic proof tests such that an objective function, such as the total cost or utility, is minimized.

The concept of the expected (statistical average) cost of failure in service has been used to obtain the optimal proof load level and the optimal thickness for spacecraft pressure vessels which are subjected to a single proof test on the ground prior to the mission [7]. It has also been applied to the optimum design of other structures subjected to a single proof test prior to service [8]. Furthermore, the concept of the expected cost of failure is applicable to the optimization of inspection frequency for metallic aircraft structures [20]. Such a concept will be employed herein for the optimization of the periodic proof test.

The objective function to be minimized in order to obtain the optimal proof load level and the optimal number of periodic proof tests is the total expected (statistical average) cost. The total expected cost, EC_T^*, considered herein consists of three parts (i) the expected cost of performing periodic proof tests, (ii) the expected

cost of components to be destroyed by periodic proof tests and (iii) the expected cost of component failure in service, including possible loss of the component itself, mission degradation, loss of lives and equipments, nonavailability for service, etc.,

$$EC^{*} = C_1[N + \overline{I}(r_0,N)] + C_2\overline{I}(r_0,N) + C_f P_f(r_0,N) \tag{34}$$

in which

C_1 = cost of performing one proof test for one component

N = total number of scheduled periodic proof tests in the design service life

$\overline{I}(r_0,N)$ = expected (statistical average) number of components to be destroyed by and subsequently replaced after the periodic proof tests in the design service life. It is a function of the proof load level, r_0, and the number, N, of the scheduled periodic proof tests.

C_2 = cost of one component

C_f = cost of component failure in service, including mission degradation, loss of component itself, possible loss of lives and properties, etc.

$P_f(r_0,N)$ = probability of failure in the design service life, which is a function of the proof load level, r_0, and the number, N, of scheduled periodic proof tests.

The expected cost of performing periodic proof tests, $C_1[N+\overline{I}(r_0,N)]$, is proportional to the total expected number of proof tests. The total expected number of proof tests consists of the number, N, of scheduled periodic proof tests plus the expected (average) number, $\overline{I}(r_0,N)$, of components to be destroyed by proof tests, since when a component is destroyed by the proof test, a new component is manufactured and proof-tested for replacement.

The second term in Eq. 34, $C_2\overline{I}(r_0,N)$, represents the total expected cost of components destroyed by periodic proof tests. The third term, $C_f P_f(r_0,N)$, is the expected cost of component failure in service.

Dividing Eq. 34 by C_f, one obtains

$$EC = \gamma_1[N + \overline{I}(r_0,N)] + \gamma_2\overline{I}(r_0,N) + P_f(r_0,N) \tag{35}$$

in which

$$EC = EC^*/C_f \tag{36}$$

is the relative expected cost (or risk function) and

$$\begin{aligned} \gamma_1 &= C_1/C_f \\ \gamma_2 &= C_2/C_f \end{aligned} \tag{37}$$

In Eq. 37, γ_1 is the ratio of the cost of performing one proof test for one component to the cost of component failure in service. γ_2 is the ratio of the cost of one component to the cost of component failure in service. Both γ_1 and γ_2 are nondimensional and they represent the relative importance, respectively, of the cost of proof test and of the cost of component itself with respect to the cost of component failure in service. They are important input parameters in the optimization process and have to be estimated subjectively or objectively if possible.

The estimation of the probability of failure, $P_f(r_0,N)$, in service has been discussed in Section IV. It has been shown that $P_f(r_0,N)$ is a monotonically decreasing function of r_0 and N. The expected number, $\overline{I}(r_0,N)$, of components to be destroyed by periodic proof tests will be derived in the next section. It will be shown that $\overline{I}(r_0,N)$ is a monotonically increasing function of r_0 and N.

The optimization problem considered herein is to find the optimal number, $\underline{N}$, of the periodic proof tests and the optimal proof load level, $\underline{r}_0$, so that the objective function EC, the relative expected cost (or risk function), is minimized, while the probability of failure of the component should be less than a specified allowable level P_a. The optimization problem can be stated as follows;

minimize the objective function EC

$$EC = \gamma_1[N + \overline{I}(r_0,N)] + \gamma_2\overline{I}(r_0,N) + P_f(r_0,N) \tag{38}$$

subject to the constraint

$$P_f(r_0,N) \leq P_a \tag{39}$$

where P_a is the allowable level of failure probability.

The mathematical techniques are available for solving the optimal solution r_0 and N for the system of equations given by Eqs. 38 and 39, for instance the method of feasible direction [26] and the nonlinear programming [27]. These classical techniques will not be discussed and the interested reader is referred to, e.g., Refs. 26 and 27.

SECTION VII

EXPECTED NUMBER, $\overline{I}(r_0,N)$, OF COMPONENTS TO BE DESTROYED BY PERIODIC PROOF TESTS

Let L be the number of newly manufactured components (prior to service) failed under the proof load r_0 before a component that survives the proof load is obtained. Obviously, L is a statistical variable. The statistical distribution of L can be shown to follow a negative Binomial distribution,

$$P[L = \ell] = (1 - B_0)B_0^{\ell} \quad ; \ell = 0, 1, 2, \cdots \tag{40}$$

in which B_0 is the probability of failure of the new component under the proof load r_0, which can be obtained from Eq. 1

$$B_0 = P[R_0 \leq r_0] = 1 - \exp\{-(r_0/\beta)^{\alpha}\} \tag{41}$$

and $(1-B_0)$ is the probability of surviving r_0.

The expected (statistical average) number, $\overline{L}$, of new components failed under the proof load r_0 before the one that survives r_0 is obtained, can be computed by use of Eq. 40

$$\overline{L} = \sum_{\ell=0}^{\infty} \ell(1-B_0)\, B_0^{\ell} = \frac{B_0}{1-B_0} \tag{42}$$

Let $\overline{I}_j$ $(j=1,2,3\cdots, N)$ be the expected (statistical average) number of components destroyed by the jth proof test performed at $(j-1)T$ flights. Then, the expected number of components destroyed under the initial (first) proof test prior to service, $\overline{I}_1$, is obvious

$$\overline{I}_1 = \overline{L} = \frac{B_0}{1-B_0} \tag{43}$$

The expected number of components failed under the second proof test performed at T is attributed to two different populations; (i) the original component (ii) the new component manufactured at T, when the original component fails under the 2nd proof test with probability B_1. Hence,

$$\bar{I}_2 = B_1 + B_1 \left[\frac{B_0}{1-B_0}\right] \tag{44}$$

in which B_1 is the probability that the original component will fail under the second proof test performed at T and is given by Eqs. 6-7. Since we are concerned with one component $B_1 \times 1$ is the expected number of the original component to be destroyed by the 2nd proof test. $B_0/(1-B_0)$ is the expected number of failures for the renewal component manufactured at T, when the original component fails (with probability B_1).

Eq. 44 can be written as

$$\bar{I}_2 = \frac{B_1}{1-B_0} \tag{45}$$

The expected number of components to be destroyed by the 3rd proof test performed at 2T comes from three different populations; (i) the original component which survives the 2nd proof test, (ii) the renewal component manufactured at 2T when the original component fails under the 3rd proof test (with probability B_2), and (iii) the renewal component manufactured at T. These three contributions are given, respectively, in the following;

$$\bar{I}_3 = B_2 + B_2 \left[\frac{B_0}{1-B_0}\right] + B_1\bar{I}_2 \tag{46}$$

in which B_2 is given by Eq. 6 for j=2.

The first term $B_2 \times 1$ is the expected number of failures attributed to the original component and the second term is the expected number of failures attributed to the renewal component manufactured at 2T when the original component fails at 2T (with probability B_2). The expected number of failures attributed to the renewal component manufactured at T is $\bar{I}_2$ and the probability of having such a renewal component is B_1. Hence $B_1\bar{I}_2$ is the contribution from the renewal component manufactured at T.

Eq. 46 can be written as

$$\bar{I}_3 = \frac{B_2}{1-B_0} + B_1\bar{I}_2 \tag{47}$$

in which $\bar{I}_2$ is given by Eq. 45.

The expected number of components, $\bar{I}_4$, to be destroyed by the 4th proof test performed at 3T is contributed by 4 different populations; (i) the original component which has survived all the previous proof tests (ii) the renewal component manufactured at 3T when the original component fails under the 4th proof test at 3T (with probability B_3), (iii) the renewal component manufactured at T for replacing the original component (with probability B_1) and (iv) the renewal component manufactured at 2T for replacing the original component (with probability B_2). Thus,

$$\begin{aligned}\bar{I}_4 &= B_3 + B_3\left[\frac{B_0}{1-B_0}\right] + B_1\bar{I}_3 + B_2\bar{I}_2 \\ &= \frac{B_3}{1-B_0} + B_1\bar{I}_3 + B_2\bar{I}_2\end{aligned} \tag{48}$$

in which B_3 is given by Eq. 6 for j=3.

In a similar manner, it can be shown that the expected number of components to be destroyed by the jth proof test is

$$\bar{I}_j = \frac{B_{j-1}}{1-B_0} + \delta_{j-2}\sum_{k=1}^{j-2} B_k\bar{I}_{j-k} \tag{49}$$

in which $\delta_{j-2}=0$ for $j \leq 2$ and $\delta_{j-2}=1$ for $j>2$.

The total expected number of components to be destroyed by periodic proof tests in the design service life (0,NT) is therefore,

$$\bar{I}(r_0,N) = \sum_{j=1}^{N} \bar{I}_j \tag{50}$$

Substitution of Eq. 49 into Eq. 50 yields

$$\bar{I}(r_0,N) = \sum_{j=1}^{N} \frac{B_{j-1}}{1-B_0} + \sum_{j=3}^{N} \sum_{k=1}^{j-2} B_k \bar{I}_{j-k} \tag{51}$$

where $\bar{I}_{j-k}$ is given by Eq. 49. Note that both B_0 and B_j are functions of r_0, and hence $\bar{I}(r_0,N)$ given by Eq. 51 is a function of r_0 and N.

The solution for $\bar{I}(r_0,N)$ derived in Eq. 51 holds for the renewal policy, i.e., a new component is manufactured and proof-tested to replace the component destroyed by proof tests. For the non-renewal policy, the solution for $\bar{I}(r_0,N)$ given by Eq. 51 still holds except that the second term (double sum) should be disregarded, since it represents the expected number of failures under proof tests due to the renewal components.

It should be mentioned that the expected (statistical average) number, $\bar{I}(r_0,N)$, of components to be destroyed by periodic proof tests derived in Eq. 51 is for one airplane (or one component). When a fleet of m airplanes (or components) is considered, the expected number of components in a fleet to be destroyed by the proof tests is $m\bar{I}(r_0,N)$. As a result, $100\bar{I}(r_0,N)$ represents the expected (statistical average) percentage of the total components in a fleet to be destroyed by the proof tests. Furthermore, under the renewal policy $100\bar{I}(r_0,N)$ is also the <u>expected percentage of replacement</u> for components destroyed by the proof tests.

SECTION VIII

NUMERICAL EXAMPLES FOR OPTIMAL PERIODIC PROOF TEST

Example 3: A Bonded Joint of Fighter Aircraft Wing

The same numerical example for a boron/titanium bonded joint of fighter aircraft as given in Example 1 is considered. In the design service life of 1500 flights, the expected percentage, $100\bar{I}(r_0,N)$ (Eq. 51), of components to be destroyed by periodic proof tests is plotted in Fig. 10 under the renewal policy. It is observed that $\bar{I}(r_0,N)$ increases monotonically as the proof load level, r_0, or the number, N, of proof tests increases.

In Fig. 10, the expected percentage of components to be destroyed by proof tests exceeds 100% for r_0 = 100 kips and $N \geq 3$. This is not surprising under the renewal policy, since a new component is manufactured and proof-tested to replace the component which fails under the proof test. It is due to such a renewal policy that the expected percentage of replacement, $100\bar{I}(r_0,N)$ (or the expected percentage of components to be destroyed by proof tests) in the design service life, may be greater than 100 percent when the proof load level or the number of proof tests is high. Under the non-renewal policy, $100\bar{I}(r_0,N)$ is bounded by 100 percent (see the first term of Eq. 51). For instance, $\bar{I}(r_0,N)=1$ under the non-renewal policy indicates that all the components are expected to be destroyed by proof tests and no airplane will be available for service by the end of the design service life. Under the renewal policy, $\bar{I}(r_0,N)=1$ indicates that by the end of the design service life, all the components which are put into service initially at t=0, referred to as the original components, are expected to be replaced by the new components manufactured later, referred to as the renewal components.

The relative expected (average) cost EC (Eq. 38) is plotted in Figs. 11(a)-11(c) as solid curves for different values of γ_1 and γ_2. Also plotted in these figures as dashed curves are the probability of failure, $P_f(r_0,N)$, of composites in the design service life. The characteristic ultimate strength β is 109 kips. Several interesting observations are given in the following;

(i) For a chosen proof load level r_0 and given values of γ_1 and γ_2, there is an optimal number, $\underline{N}$, for the periodic proof test at which the relative expected cost EC is minimum. This is observed from Fig. 11 that each solid curve has a minimum as indicated by a circle.

(ii) The optimal number, $\underline{N}$, of the periodic proof test for a chosen r_0 increases as the values of γ_1 and γ_2 decrease, indicating

that as the costs of proof testing and replacement decrease, the optimal number of the periodic proof test increases. This is consistent with our intuition that the less expensive the cost of proof testing and the cost of components are, the more one can afford to perform more proof tests to achieve higher level of reliability for composites.

For a chosen proof load level r_0 and given values of γ_1 and γ_2, the local minimum as indicated by a circle may or may not be the optimal solution depending on the constraint of the allowable probability of failure P_a given by Eq. 39. Since the probability of failure, $P_f(r_0,N)$, is also plotted in Fig. 11, it can easily be determined whether the local minimum is the optimum solution. For instance, if $P_a=10^{-3}$ (reliability 0.999), it is clear from Fig. 11(a) that for $r_0=100$ kips, all the minima are optimal solutions for various γ_1 and γ_2 values. In such a situation, the constraint of Eq. 39 is said to be inactive indicating that inequality holds for Eq. 39. For $r_0=95$ kips, however, all the minima are not feasible solutions, since the probabilities of failure associated with these minima are greater than P_a as indicated by Fig. 11(b). It is observed from Fig. 11(b) that the optimal solution is N=14 for some values of γ_1 and γ_2 as indicated by stars. In this situation, the solution is primarily determined by the constraint (Eq. 39), and hence the equality sign holds for Eq. 39. The constraint is said to be active.

Above discussions are based on the premise that a proof load level is chosen *a priori*. For practical applications, it may be desirable to determine both the optimal proof load level, $\underline{r}_0$, and the optimal number, $\underline{N}$, of periodic proof tests for given values of γ_1 and γ_2. To accomplish this objective, the solid curves plotted in Fig. 10 associated with a particular set of γ_1 and γ_2 values are replotted in Fig. 12. For instance, if $P_a=10^{-2}$, it is observed from Fig. 12(a) that $\underline{r}_0=95$ kips, $\underline{N}=3$ results in a minimum EC as indicated by a circle where the constraint of Eq. 39 is satisfied. On the other hand, if $P_a=10^{-3}$, the solution $r_0=95$ kips, N=3 is not feasible since the associated probability of failure is greater than 10^{-3}. The optimal solution is $r_0=100$ kips, N=2, as indicated by a square. From the observation of extensive numerical results, general trends for the optimal solution are as follows;

(i) when the cost of component is high but the cost of proof tests is not high, i.e., γ_2 is not small but γ_1 is very small, the optimal solution is in favor of low proof load level, since high proof load level tends to destroy more components as shown by Fig. 10,

(ii) when the cost of performing proof tests and the cost of replacement are low, i.e., γ_1 and γ_2 are small, the optimal solution is in favor of high proof load level and large number N of proof tests.

This is because one can afford to loose more components to achieve high level of reliability, and

(iii) when the constraint on the allowable failure probability is stringent, i.e., P_a is small, the optimal solution is in favor of the high proof load level.

Another significant observation is that the optimal solution is not critical to the proof load level, in the sense that EC is a slowly varying function of r_0. Hence, if we do not choose exactly the optimal proof load level, the increase in the relative cost EC is not significant. For instance, the optimal solution for $\gamma_1=10^{-2}$, $\gamma_2=10^{-1}$ (see Fig. 12a) is $\underline{r}_0$=95 kips, $\underline{N}$=3 with EC=0.105. If we choose r_0=100 kips, N=2, the relative cost $\overline{EC}$ is 0.11, and EC=0.12 for r_0=90 kips, N=5. This conclusion is of practical importance and is very beneficial to the planning of proof tests, since we may choose a preferable level of proof load without much penalty. On the other hand, however, the relative expected cost EC is sensitive to the change of N, the number of periodic proof tests, as indicated by each curve of Fig. 11. As a result, care should be taken in choosing the optimal number $\underline{N}$ for periodic proof tests.

Example 4: Transport-Type Aircraft

The same numerical example for a glass/epoxy $\pi/4$ laminates as given in Example 2 is considered. In the design service life of 10,000 flight hours, the expected percentage, $100\bar{I}(r_0,N)$, of components to be destroyed by periodic proof tests is plotted in Fig. 13 under the renewal policy. It is observed that $\bar{I}(r_0,N)$ increases monotonically as the proof load level, r_0, or the number, N, of periodic proof tests increases.

The relative expected cost EC is plotted in Figs. 14(a)-14(c) as solid curves for various values of proof load level, r_0. Also plotted in Fig. 14 as dashed curves are the probability of failure, $P_f(r_0,N)$, within the design service life. The characteristic ultimate strength β is 53 ksi.

The relative expected cost EC is also plotted in Fig. 15 for a particular set of γ_1 and γ_2 values. The optimal proof load level, $\underline{r}_0$, and the optimal frequency, $\underline{N}$, of periodic proof tests can be determined graphically from Figs. 14 and 15.

General trends for the optimal solution observed in the example of boron/titanium bonded joint hold for the present example. The important fact that the optimal solution is not critical to the proof load level is also shown in Fig. 15. It is observed from

Fig. 15 that the proof load level can be chosen from 40 ksi to 45 ksi (i.e., 0.75β to 0.85β) without much penalty involved. Hence a designer has certain degree of freedom to choose a preferable proof load level in this region, but care should be taken in determining the optimal number, $\underline{N}$, of periodic proof tests from Fig. 15.

SECTION IX

CONCLUSION

An exploratory reliability analysis of composites under random service loads and periodic proof tests have been presented. The composite reliability is obtained as a function of the statistics of ultimate strength, the statistics of service loads, composite strength degradation characteristics, design stresses, proof load levels, number of periodic proof tests, etc. It is demonstrated that the reliability of composites can be improved significantly by the application of periodic proof tests. It is shown that the composite reliability increases as the number of periodic proof tests increases or as the proof load level increases, provided that the proof load level is below the critical load level, r^*, which may damage the composites. Another conclusion from this study is that a low proof load level does not produce meaningful reliability improvement, unless the fleet size is large.

An exploratory study for the optimization of periodic proof tests based on cost and reliability analysis has also been presented. The optimal proof load level and the optimal number of periodic proof tests are determined by minimizing the total expected (statistical average) cost, while the constraint on the reliability of composites is satisfied. It is shown that the optimal periodic proof test is not critical to the proof load level, in the sense that if a non-optimal proof load level is chosen, the cost penalty is not significant. This conclusion is very beneficial to the planning of proof tests. However, the optimal periodic proof test is sensitive to the number, N, of periodic proof tests.

The present study concentrates on the theoretical development of both the methodologies of reliability analysis and cost optimizations for composites and the theory of periodic proof tests. Experimental verifications for the residual strength degradation model used herein as well as the theory of periodic proof tests are presented in Ref. 28.

In the numerical examples, the proof load level is restricted to be below the critical load level r* that may damage the composites. Experimental data [1-3, 28-29] indicate that the critical load level r* is above 90% of the characteristic strength β. It is believed that a proof load level higher than the critical load level, r*, is unacceptable to the design practice. Nevertheless, if there is any need to raise the proof load level beyond r*, Eq. II-5 and II-6 derived in Appendix II and III should be used where extensive test data is needed to characterize the damage functions, $D_1(r_0/\beta)$ and $D_2(r_0/\beta)$.

The premis of the design practice seems to test the components at limit load, whereas the characteristic ultimate load is 1.5 times the limit load. However, it is observed from the numerical examples that a proof load level at the limit load, which is 66.7% of the characteristic ultimate strength β, is too low to produce significant benefit as far as the reliability improvement of composites is concerned. This conclusion is of practical importance.

The present approach does not account for the environmental effect explicitly, such as temperature, humidity, stress-corrosion, aging, etc., because of lack of statistical background data. However, these important effects can be accounted for in the present analysis because they can be reflected in the parameters c and Φ associated with the strength degradation, [see Eq. 4]. As a result, c and Φ are functions of temperature, humidity, aging, etc., and have to be determined from test results under flight-by-flight real time loading history superimposed by the variations of temperature, humidity, etc. Further research is needed in this regard.

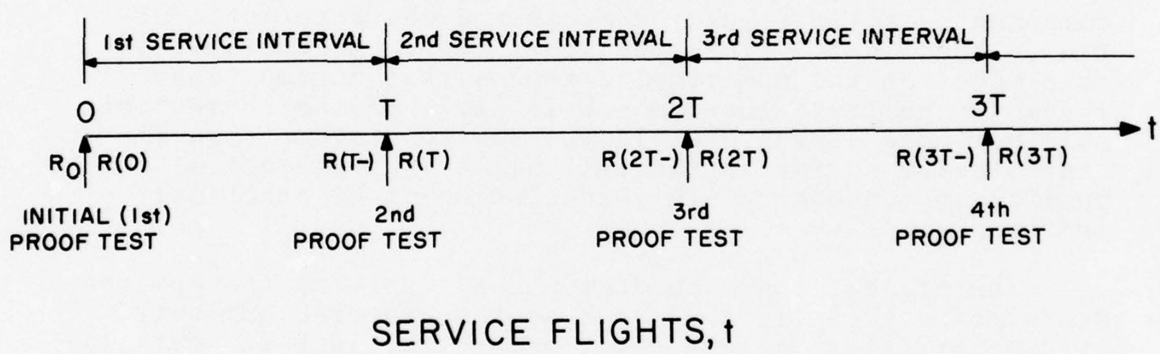

FIG. 1.: SERVICE INTERVAL AND PERIODIC PROOF TEST

PROBABILITY DENSITY FUNCTION OF
RESIDUAL STRENGTH $f_{R(t)}(x)$

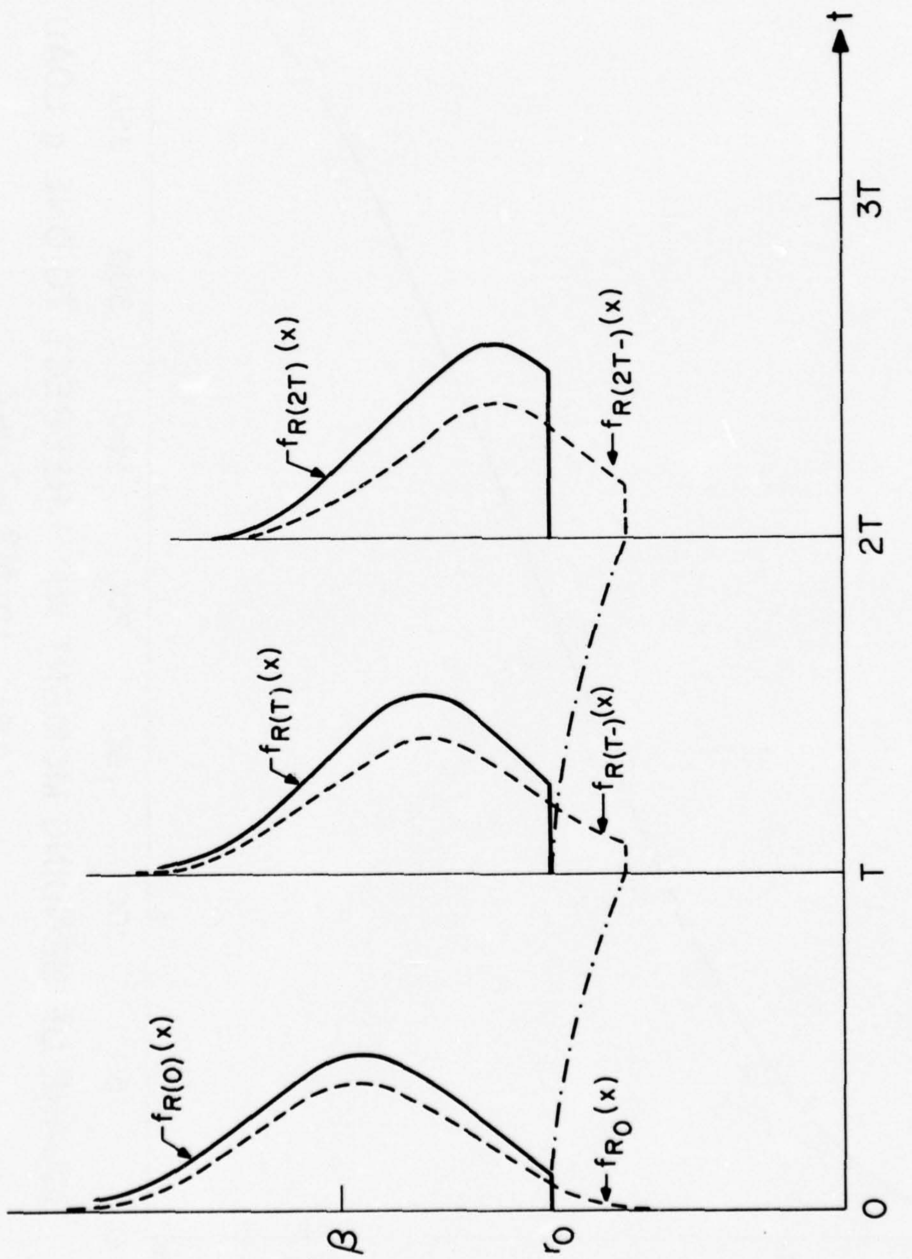

FIG. 2.: PROBABILITY DENSITY FUNCTION OF RESIDUAL STRENGTH
VS SERVICE TIME

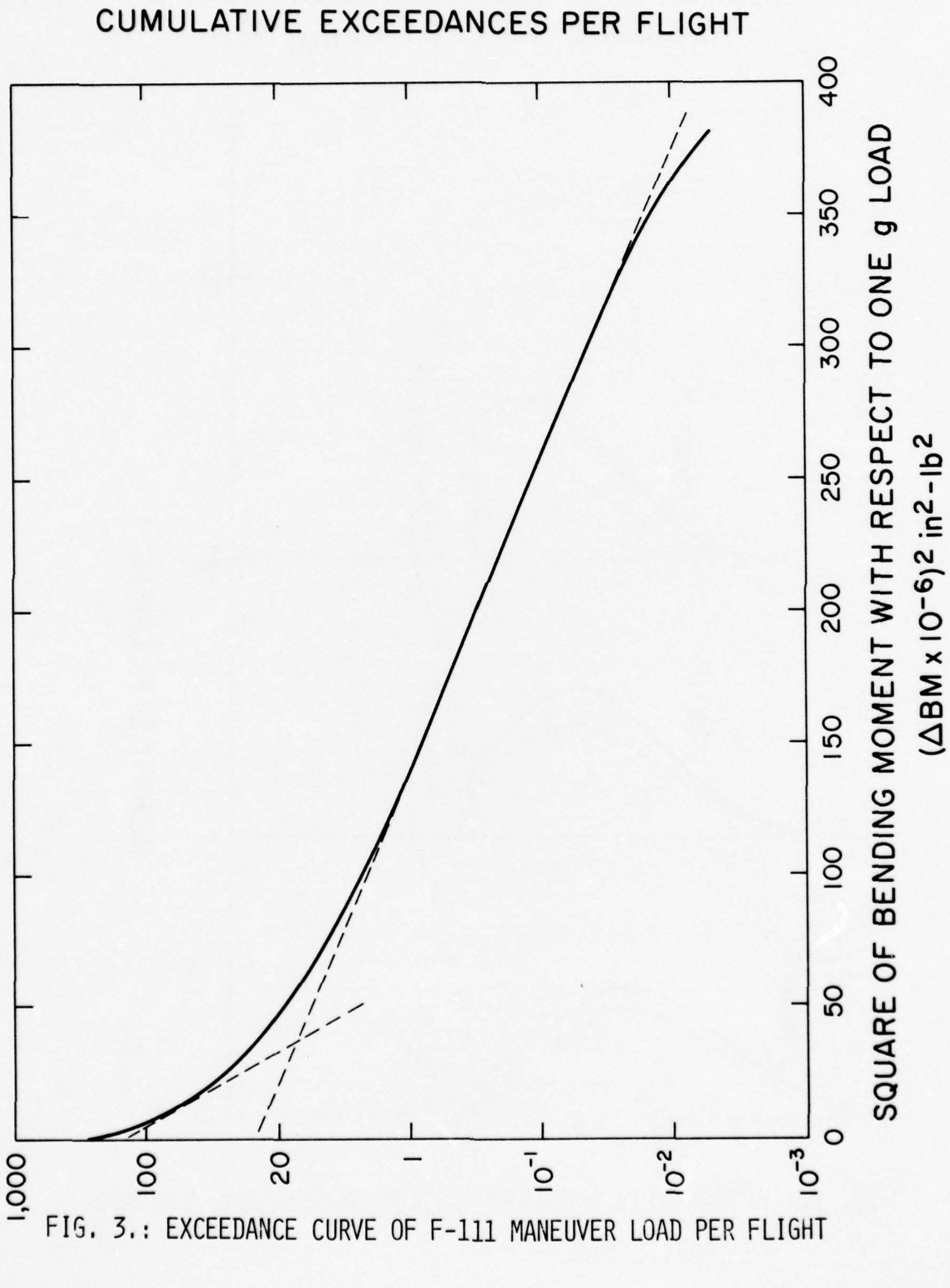

FIG. 3.: EXCEEDANCE CURVE OF F-111 MANEUVER LOAD PER FLIGHT

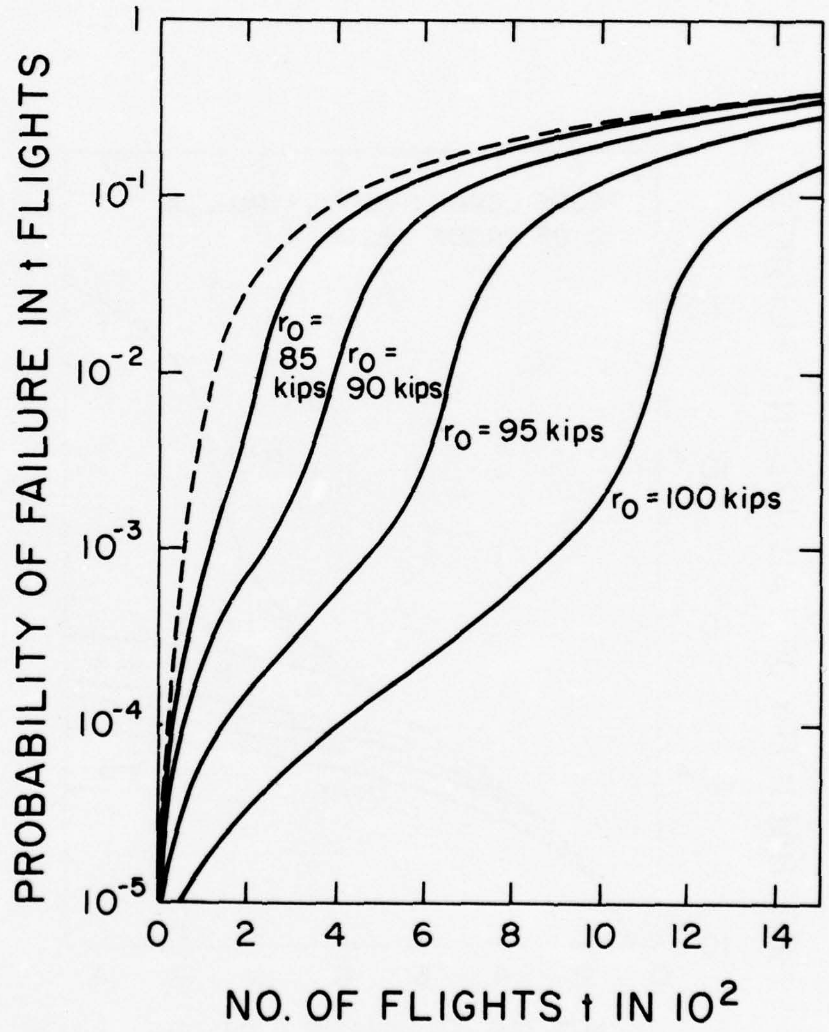

FIG. 4.: PROBABILITY OF FAILURE VS SERVICE TIME; NONE AND SINGLE PROOF TEST (β = 109 KIPS)

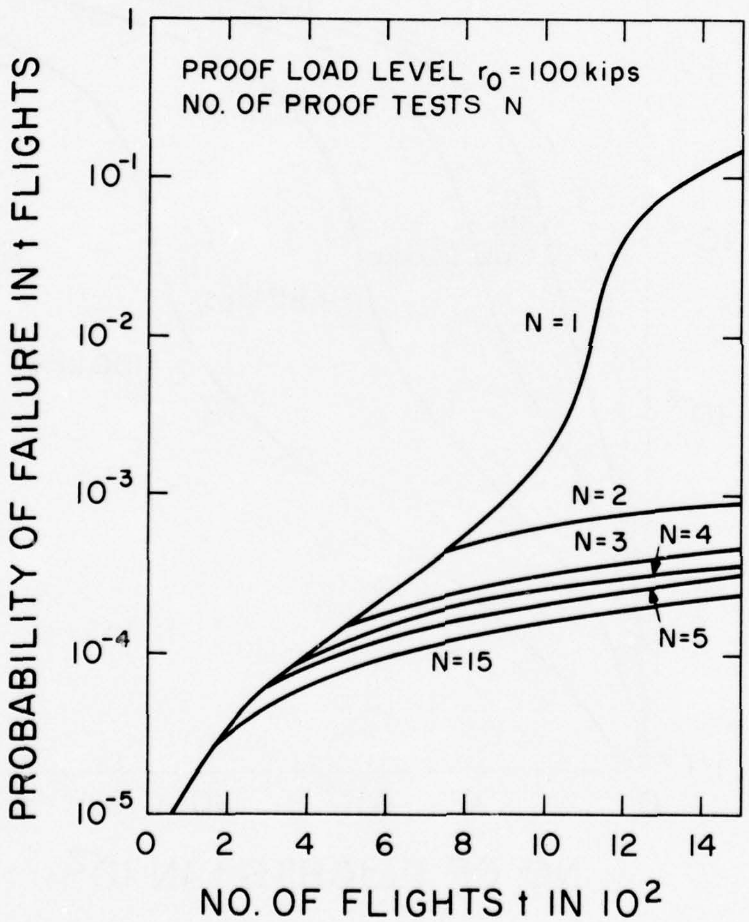

FIG. 5.: PROBABILITY OF FAILURE VS SERVICE TIME FOR VARIOUS NUMBER OF PERIODIC PROOF TESTS (BORON/TITANIUM BONDED JOINT β = 109 KIPS) (A) r_0 = 100 KIPS

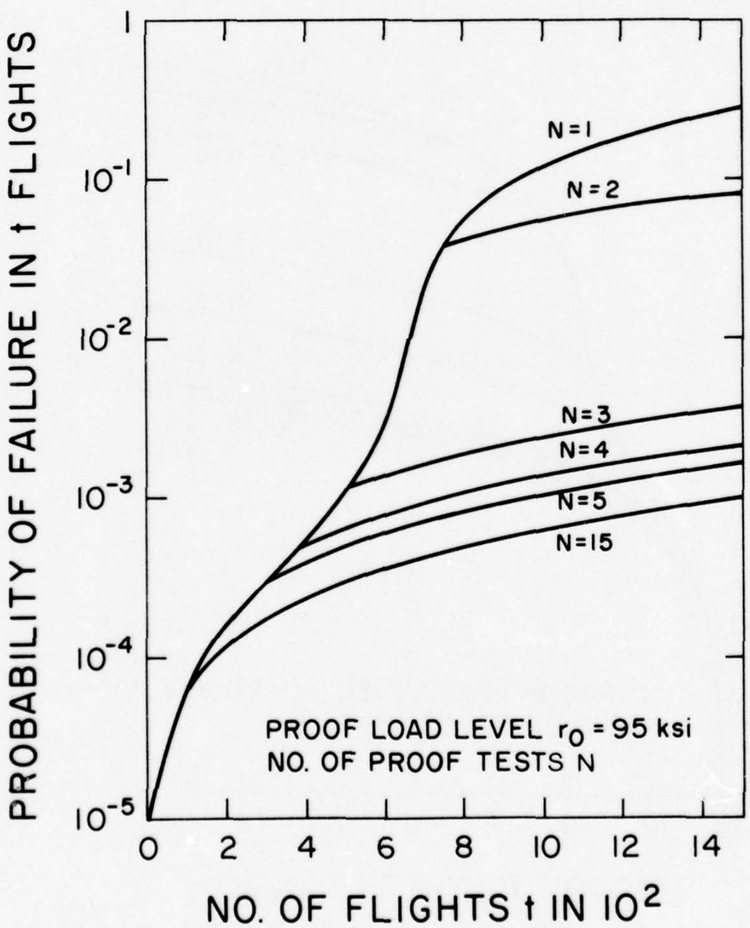

FIG. 5.: PROBABILITY OF FAILURE VS SERVICE TIME FOR VARIOUS NUMBER OF PERIODIC PROOF TESTS (BORON/TITANIUM BONDED JOINT β = 109 KIPS) (B) r_0 = 95 KIPS

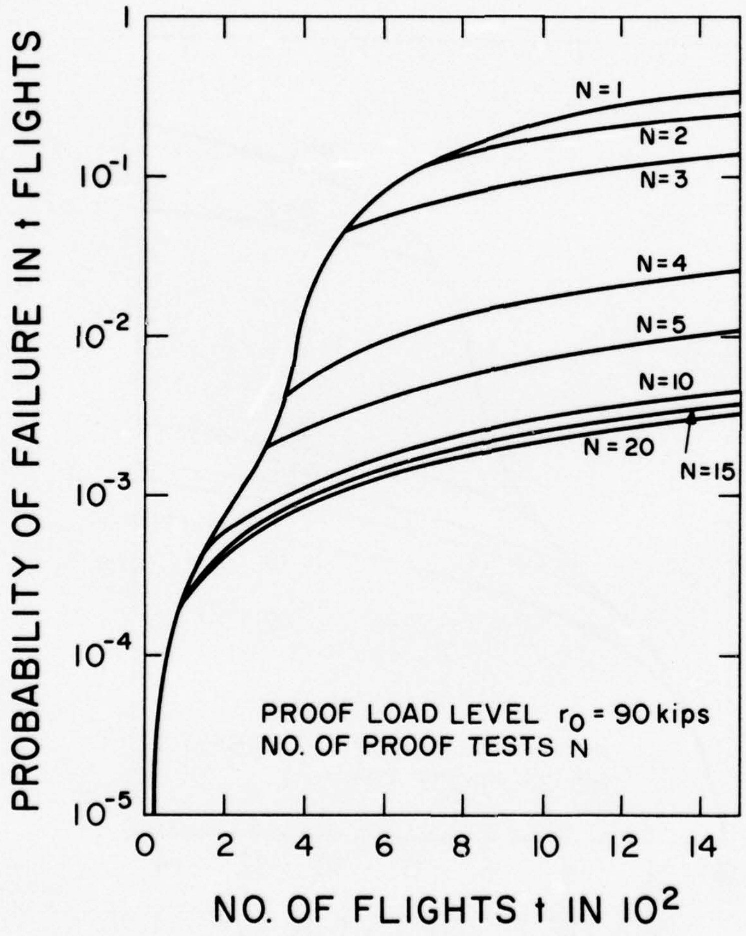

FIG. 5.: PROBABILITY OF FAILURE VS SERVICE TIME FOR VARIOUS NUMBER OF PERIODIC PROOF TESTS (BORON/TITANIUM BONDED JOINT β = 109 KIPS) (c) r_0 = 90 KIPS

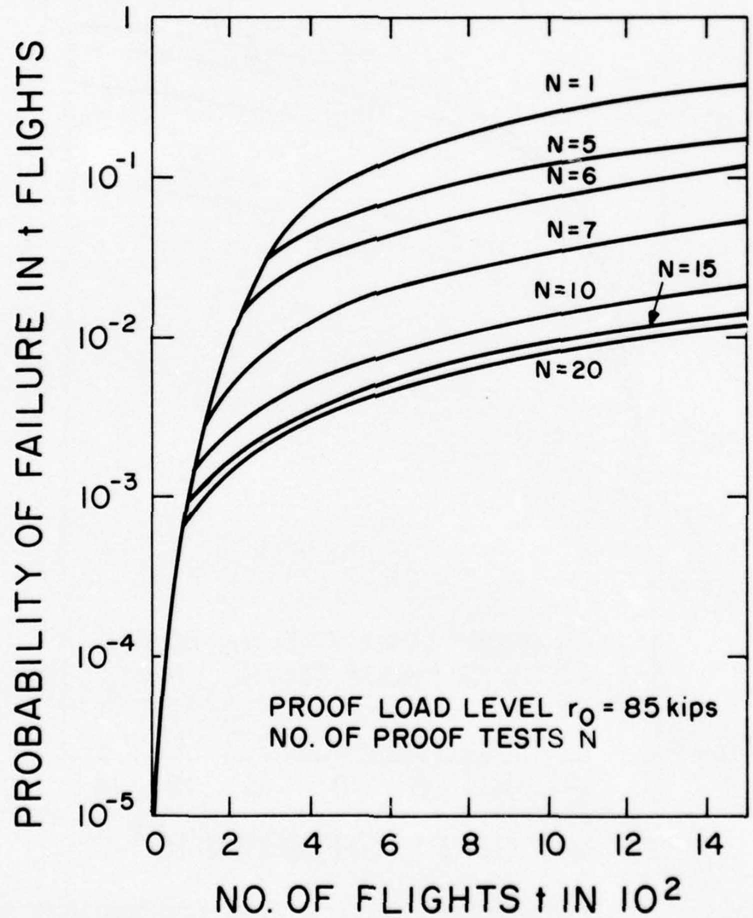

FIG. 5.: PROBABILITY OF FAILURE VS SERVICE TIME FOR VARIOUS NUMBER OF PERIODIC PROOF TESTS (BORON/TITANIUM BONDED JOINT β = 109 KIPS) (D) r_0 = 85 KIPS

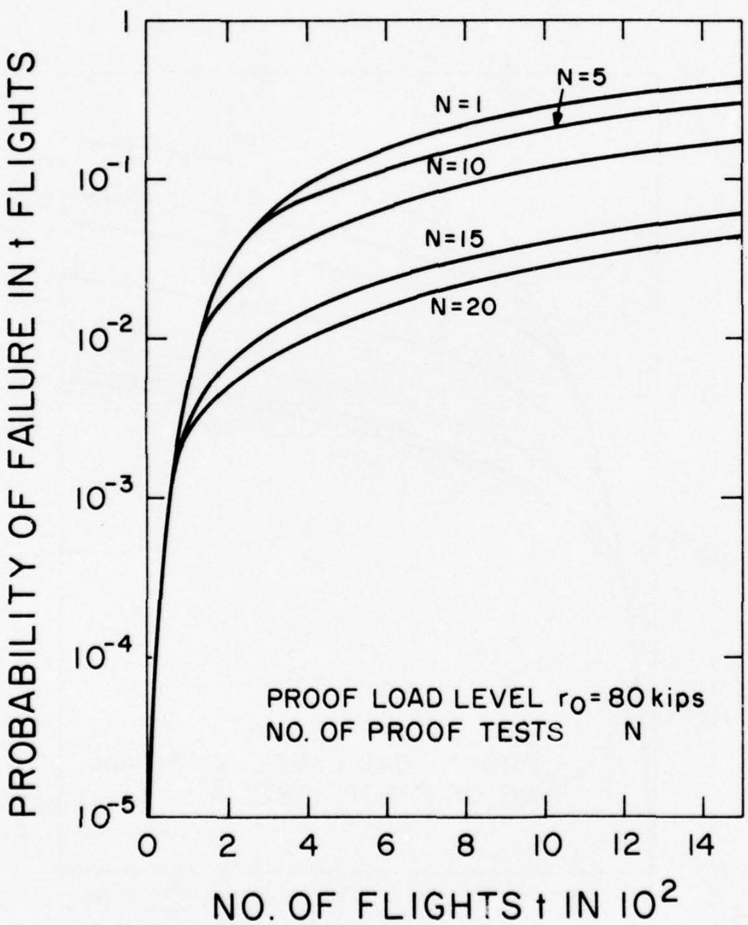

FIG. 5.: PROBABILITY OF FAILURE VS SERVICE TIME FOR VARIOUS NUMBER OF PERIODIC PROOF TESTS (BORON/TITANIUM BONDED JOINT β = 109 KIPS) (E) r_0 = 80 KIPS

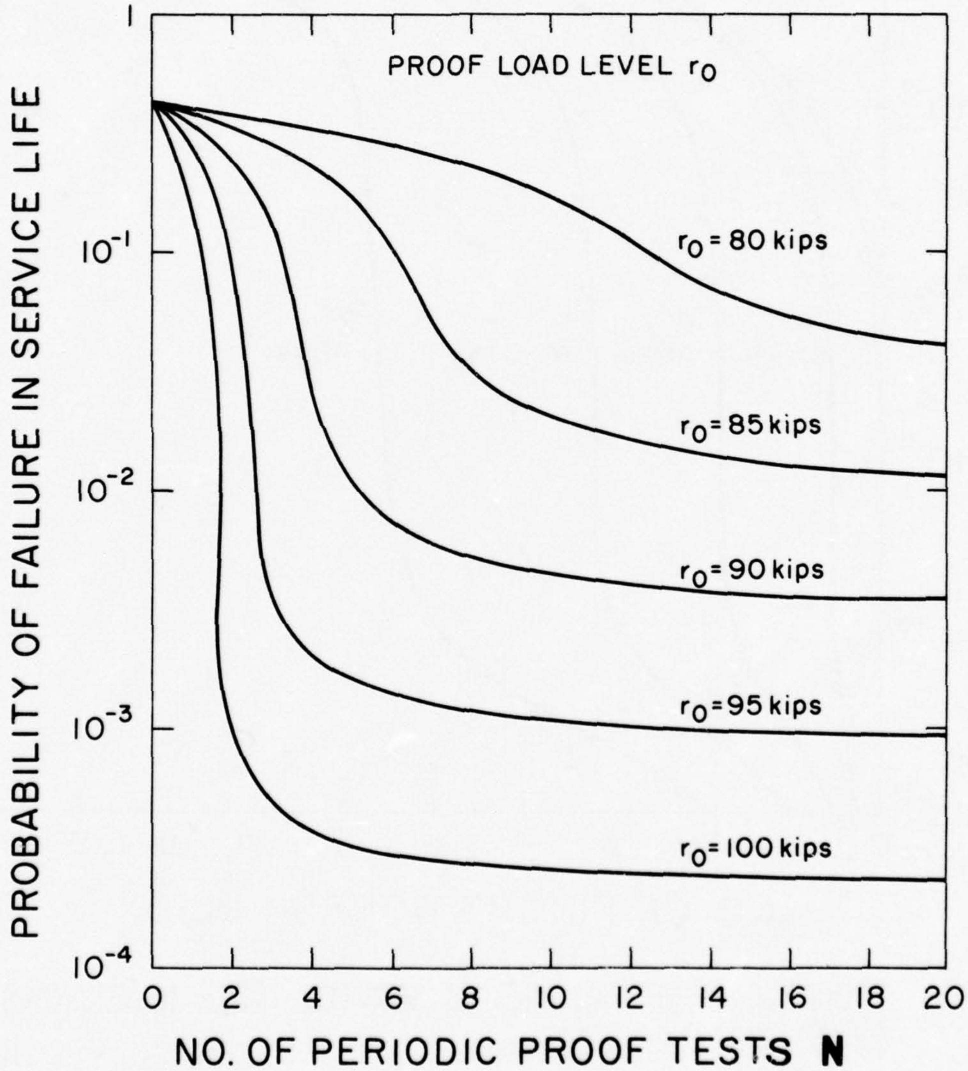

FIG. 6.: PROBABILITY OF FAILURE IN SERVICE LIFE (1500 FLIGHTS) VS PROOF LOAD LEVEL, r_0, AND NUMBER OF PERIODIC PROOF TESTS, N; (CHARACTERISTIC ULTIMATE STRENGTH β = 109 KIPS).

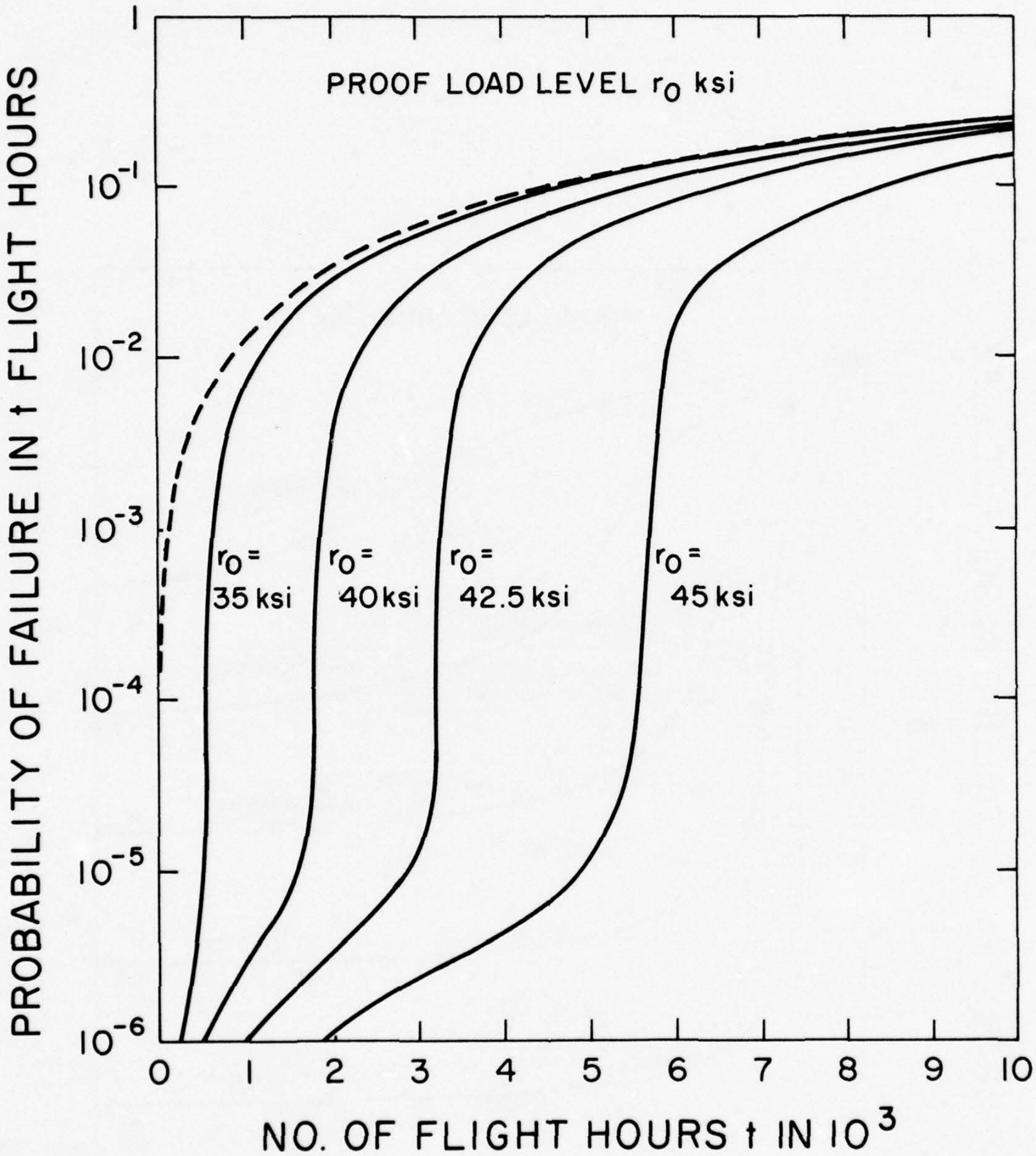

FIG. 7.: PROBABILITY OF FAILURE VS SERVICE TIME; NONE AND SINGLE PROOF TESTS (β = 53 KSI).

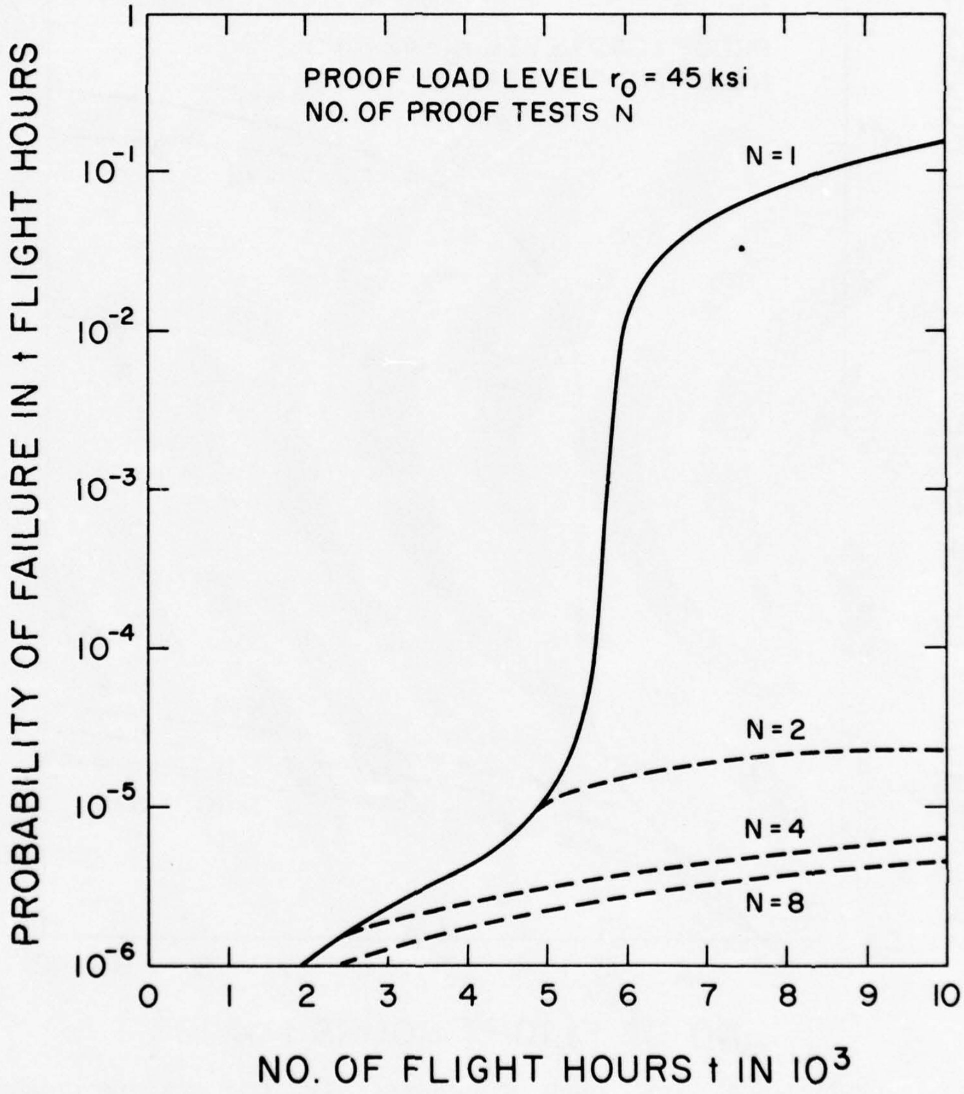

FIG. 8.: PROBABILITY OF FAILURE VS SERVICE TIME FOR VARIOUS NUMBER OF PERIODIC PROOF TESTS ($\pi/4$ GLASS/EPOXY LAMINATE β = 53 KSI): (A) r_0 = 45 KSI

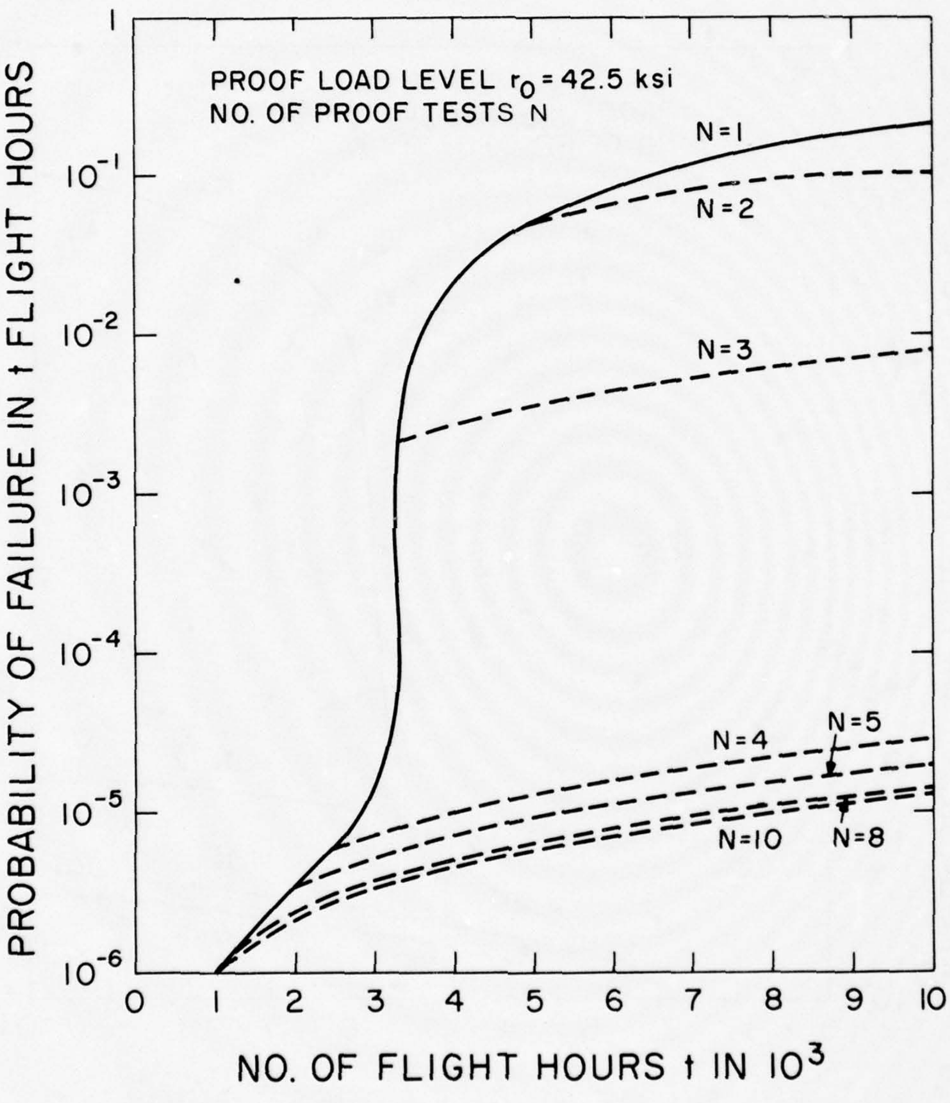

FIG. 8.: PROBABILITY OF FAILURE VS SERVICE TIME FOR VARIOUS NUMBER OF PERIODIC PROOF TESTS ($\pi/4$ GLASS/EPOXY LAMINATE β = 53 KSI): (B) r_0 = 42.5 KSI

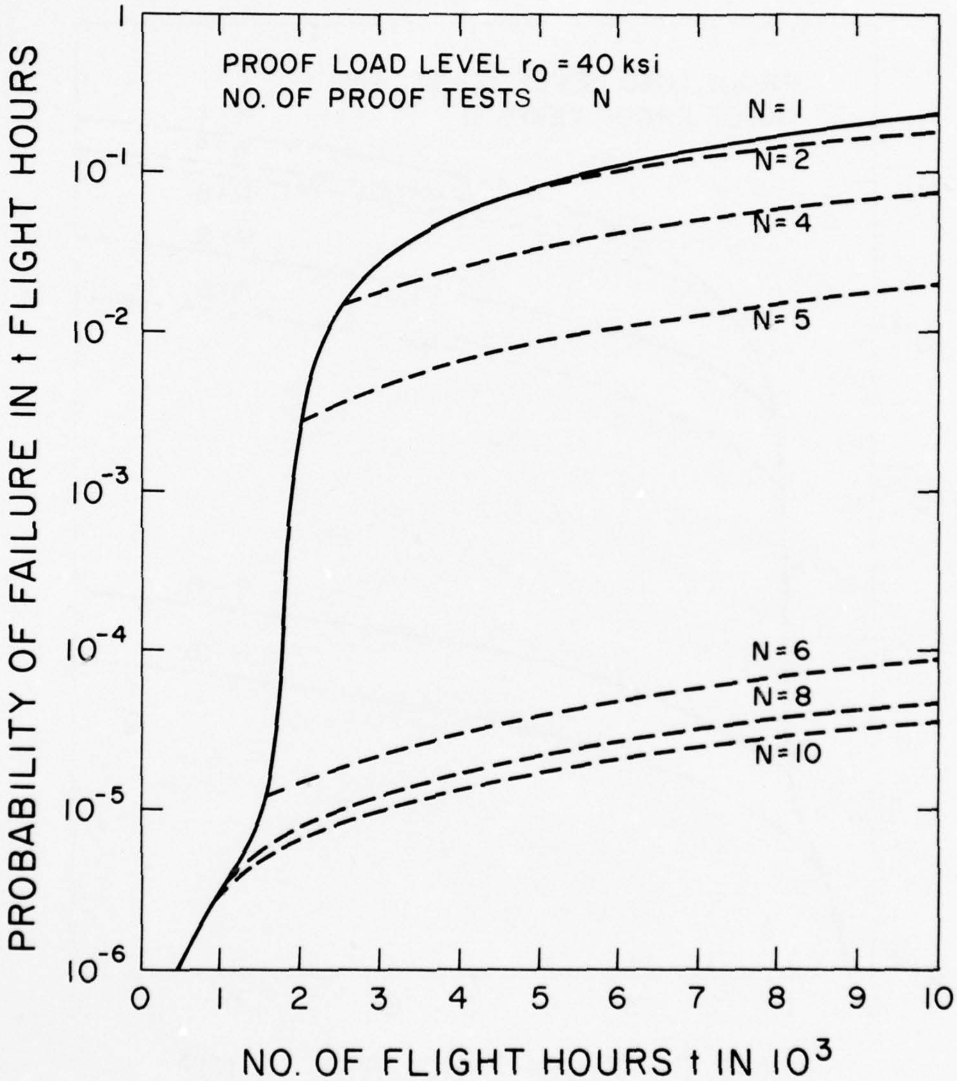

FIG. 8.: PROBABILITY OF FAILURE VS SERVICE TIME FOR VARIOUS NUMBER OF PERIODIC PROOF TESTS ($\pi/4$ GLASS/EPOXY LAMINATE β = 53 KSI): (c) r_0 = 40 KSI

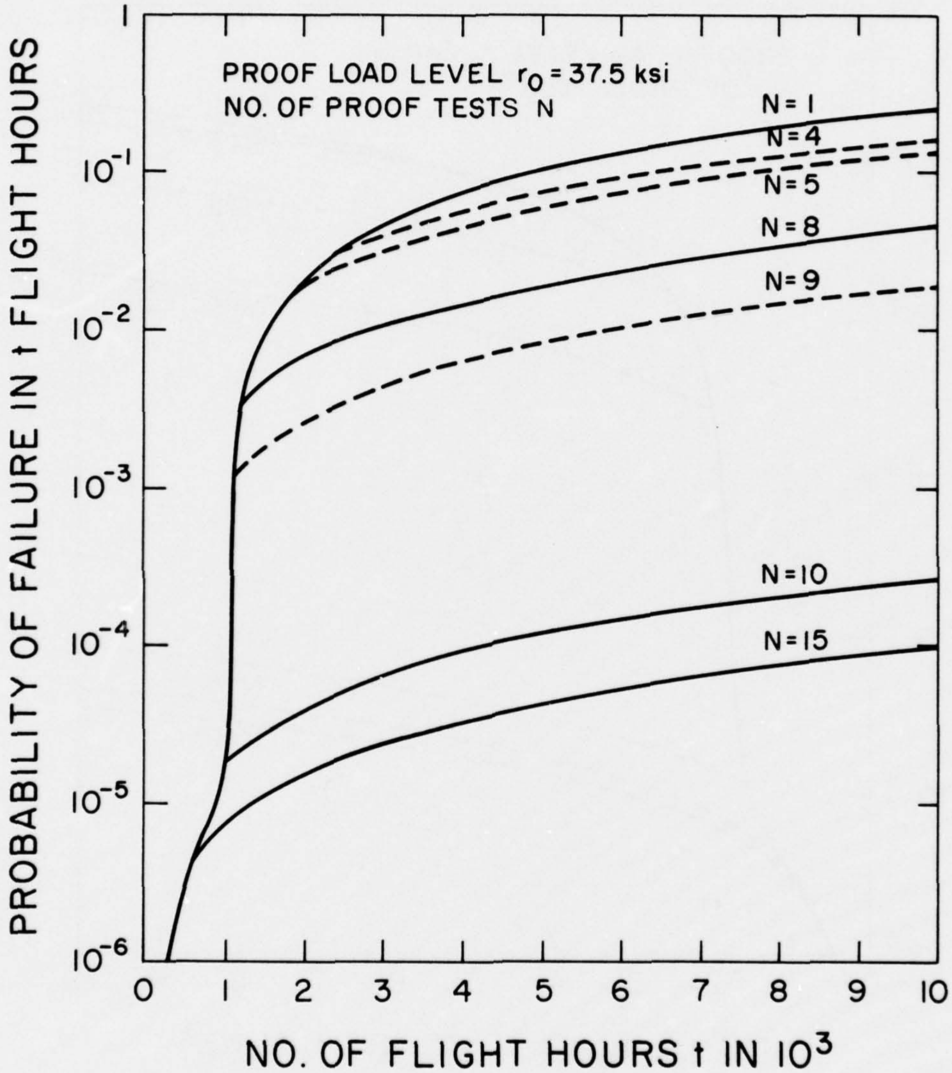

FIG. 8.: PROBABILITY OF FAILURE VS SERVICE TIME FOR VARIOUS NUMBER OF PERIODIC PROOF TESTS ($\pi/4$ GLASS/EPOXY LAMINATE β = 53 KSI): (D) r_0 = 37.5 KSI

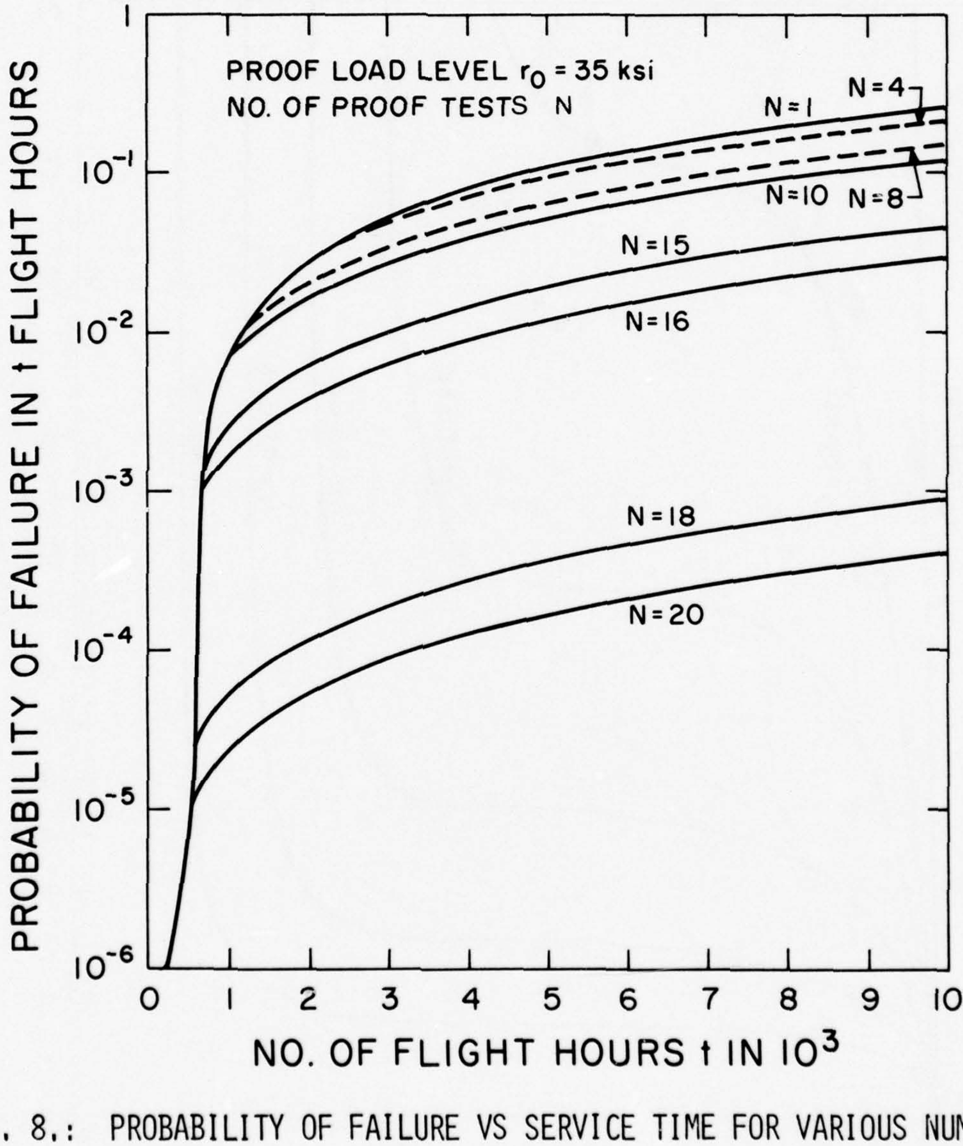

FIG. 8.: PROBABILITY OF FAILURE VS SERVICE TIME FOR VARIOUS NUMBER OF PERIODIC PROOF TESTS ($\pi/4$ GLASS/EPOXY LAMINATE β = 53 KSI): (E) r_0 = 35 KSI

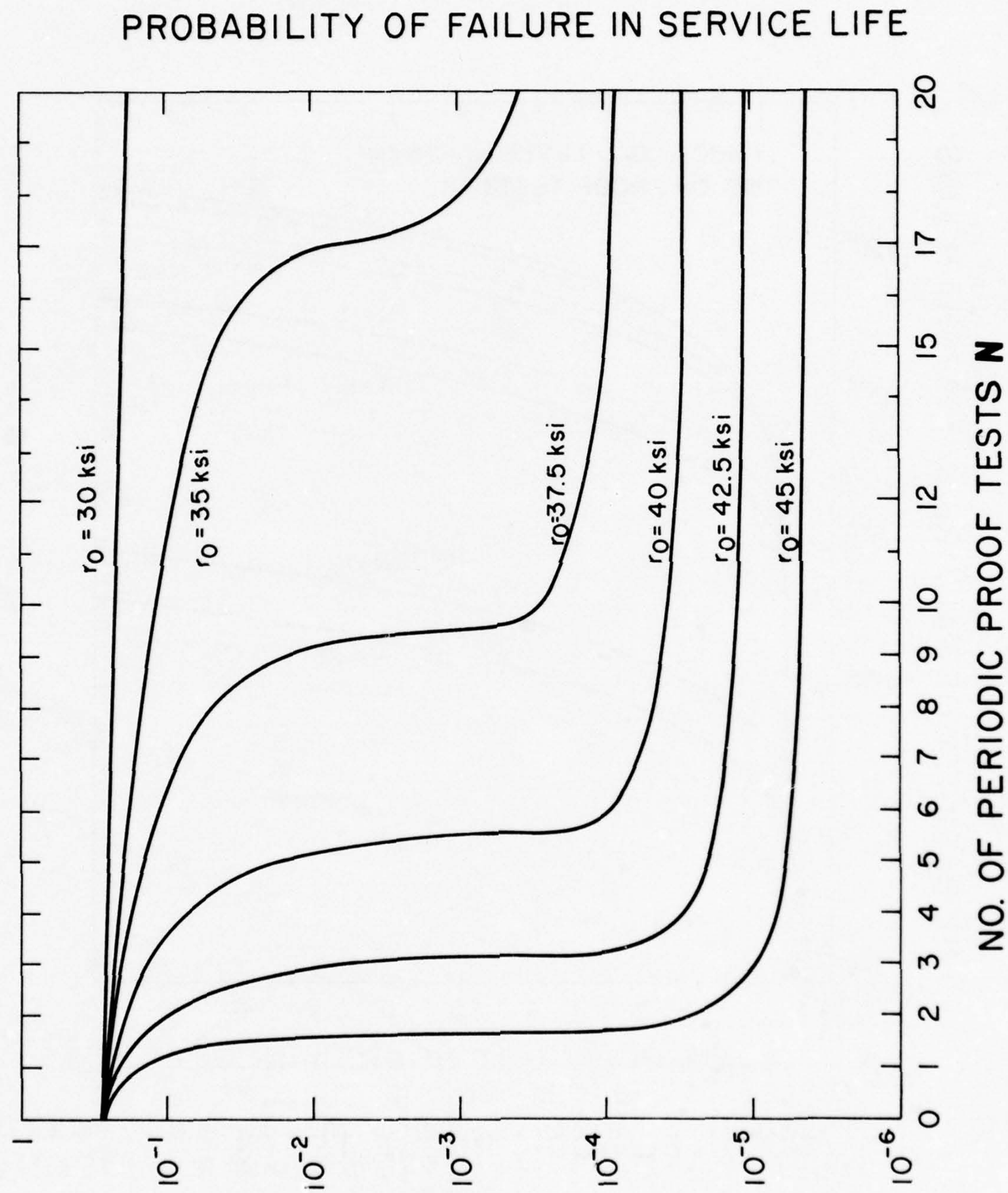

FIG. 9.: PROBABILITY OF FAILURE IN SERVICE LIFE (10,000 FLIGHT HOURS) VS PROOF LOAD LEVEL, r_0, AND NUMBER OF PROOF TESTS, N:

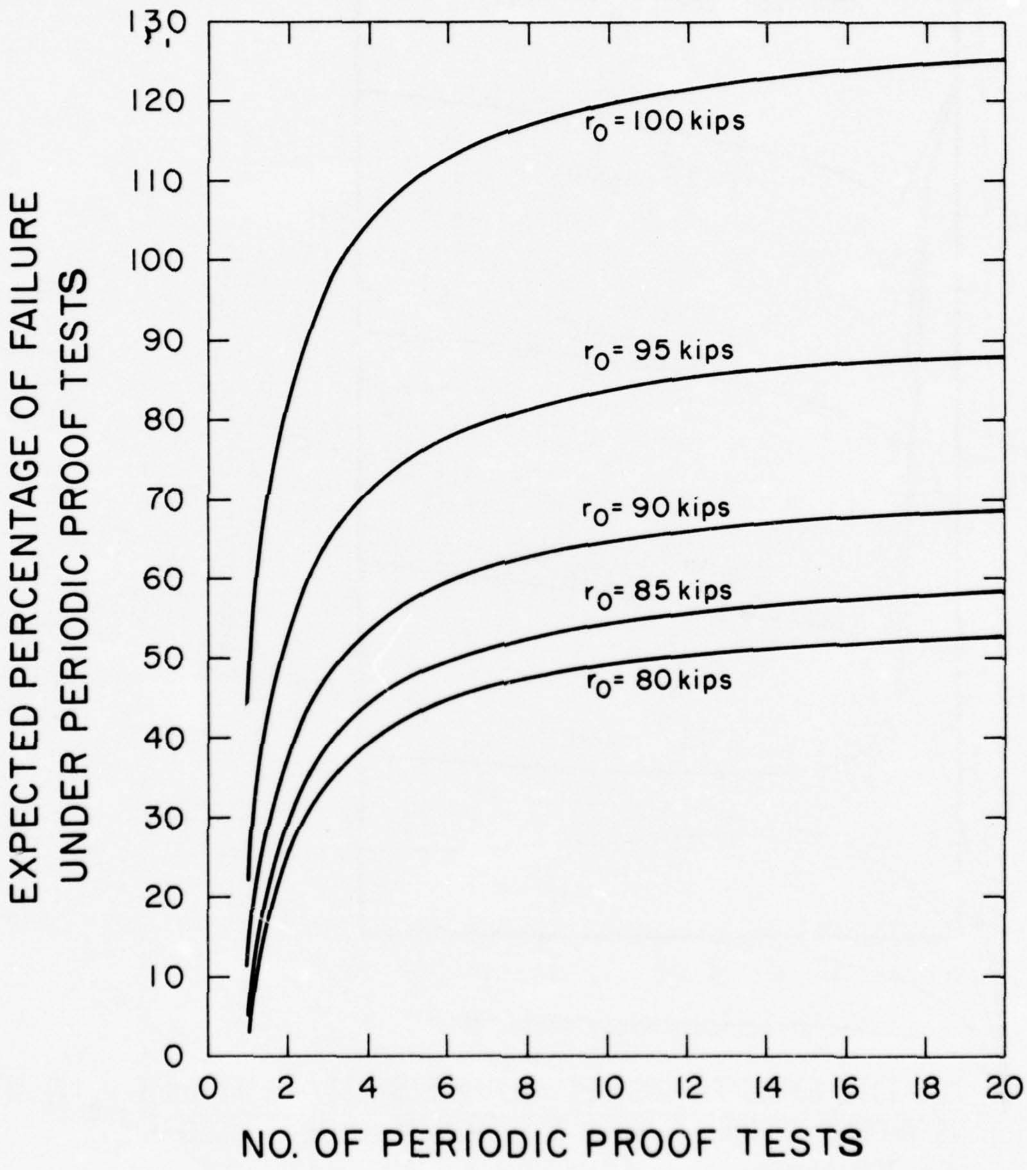

FIG. 10.: EXPECTED PERCENTAGE OF COMPONENTS TO BE DESTROYED BY PERIODIC PROOF TESTS (BORON/TITANIUM BONDED JOINT)

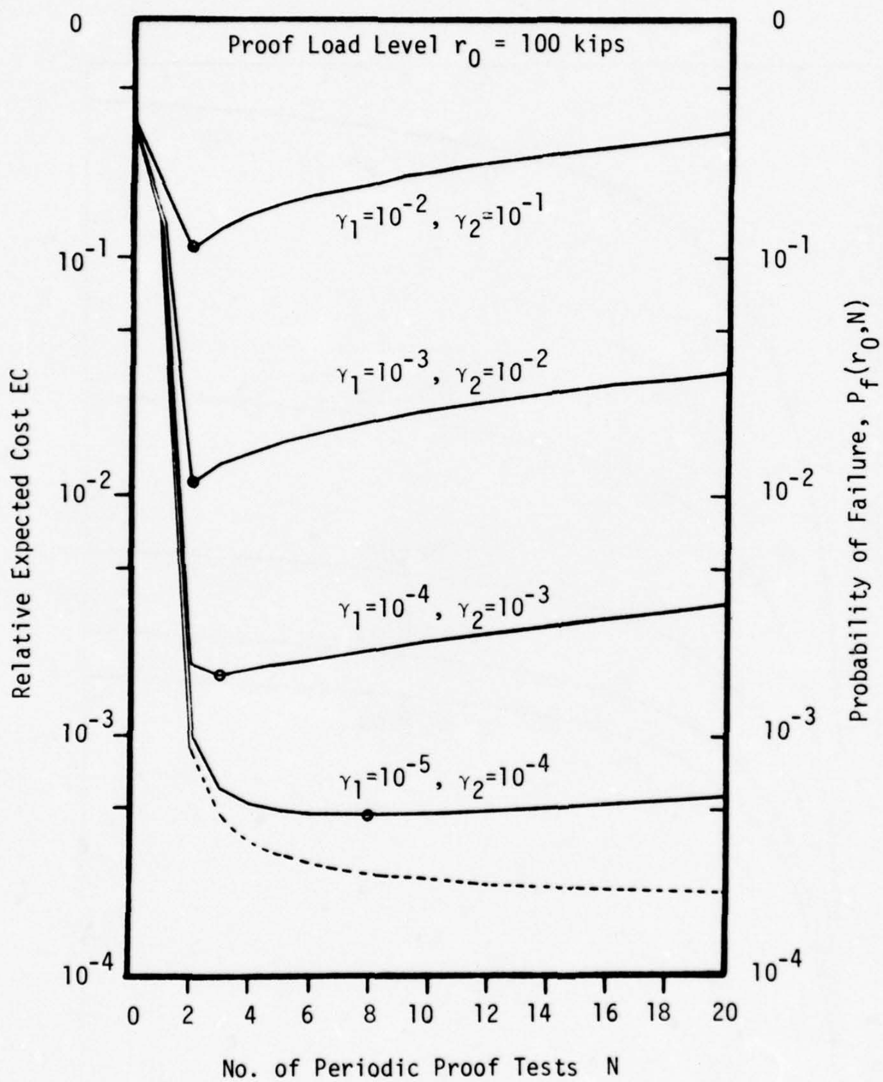

FIG. 11.: RELATIVE EXPECTED COST EC AND PROBABILITY OF FAILURE $P_F(r_0,N)$ VS NUMBER OF PROOF TESTS N (BORON/TITANIUM BONDED JOINT β = 109 KIPS):
(A) r_0 = 100 KIPS

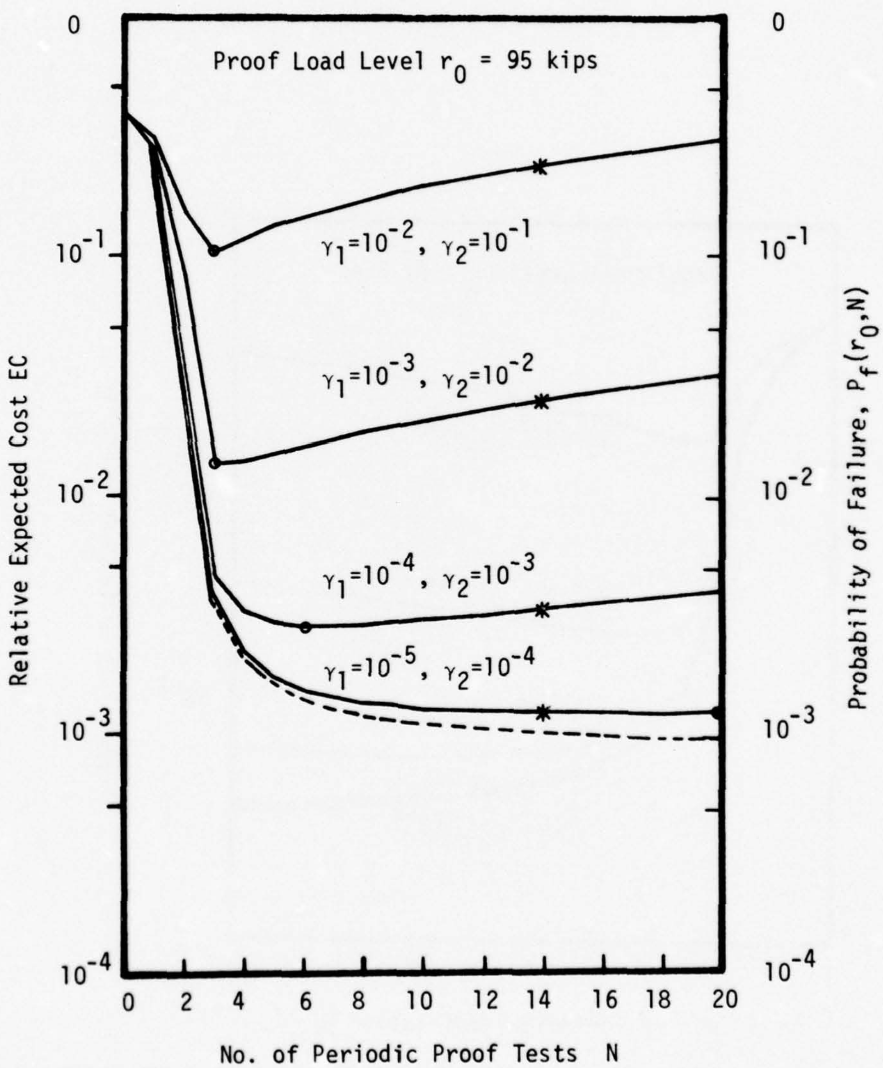

FIG. 11.: RELATIVE EXPECTED COST EC AND PROBABILITY OF FAILURE $P_F(r_0,N)$ VS NUMBER OF PROOF TESTS N (BORON/TITANIUM BONDED JOINT β = 109 KIPS):
(B) r_0 = 95 KIPS

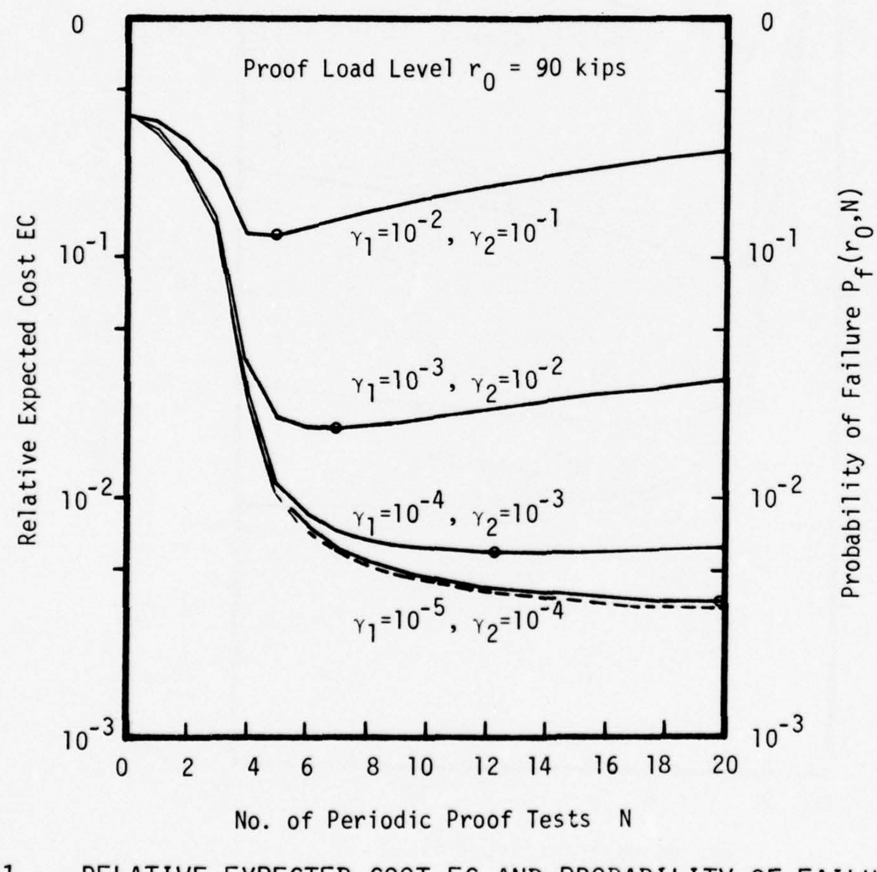

FIG. 11.: RELATIVE EXPECTED COST EC AND PROBABILITY OF FAILURE $P_F(r_0, N)$ VS NUMBER OF PROOF TESTS N (BORON/TITANIUM BONDED JOINT β = 109 KIPS):
(c) r_0 = 90 KIPS

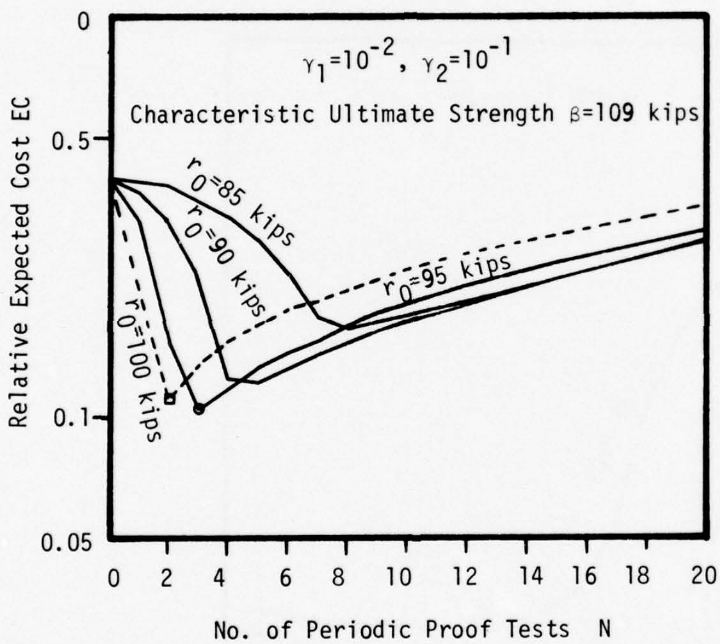

FIG. 12.: RELATIVE EXPECTED COST EC VS NUMBER OF PROOF TESTS FOR VARIOUS VALUES OF γ_1 AND γ_2:
(A) $\gamma_1 = 10^{-2}$, $\gamma_2 = 10^{-1}$

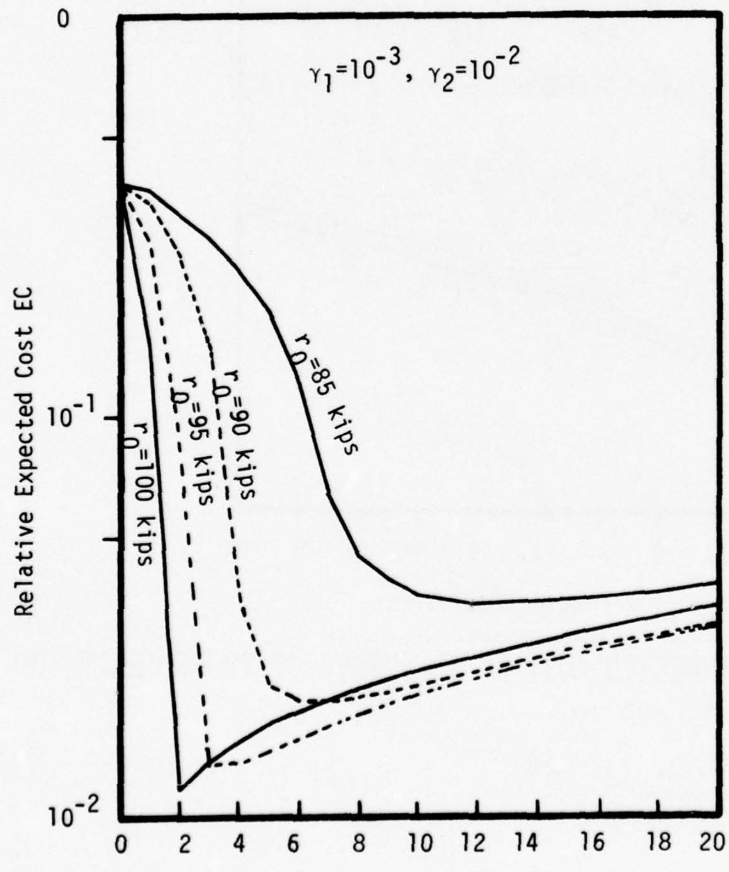

FIG. 12.: RELATIVE EXPECTED COST EC VS NUMBER OF PROOF TESTS FOR VARIOUS VALUES OF γ_1 AND γ_2:
(B) $\gamma_1 = 10^{-3}$, $\gamma_2 = 10^{-2}$

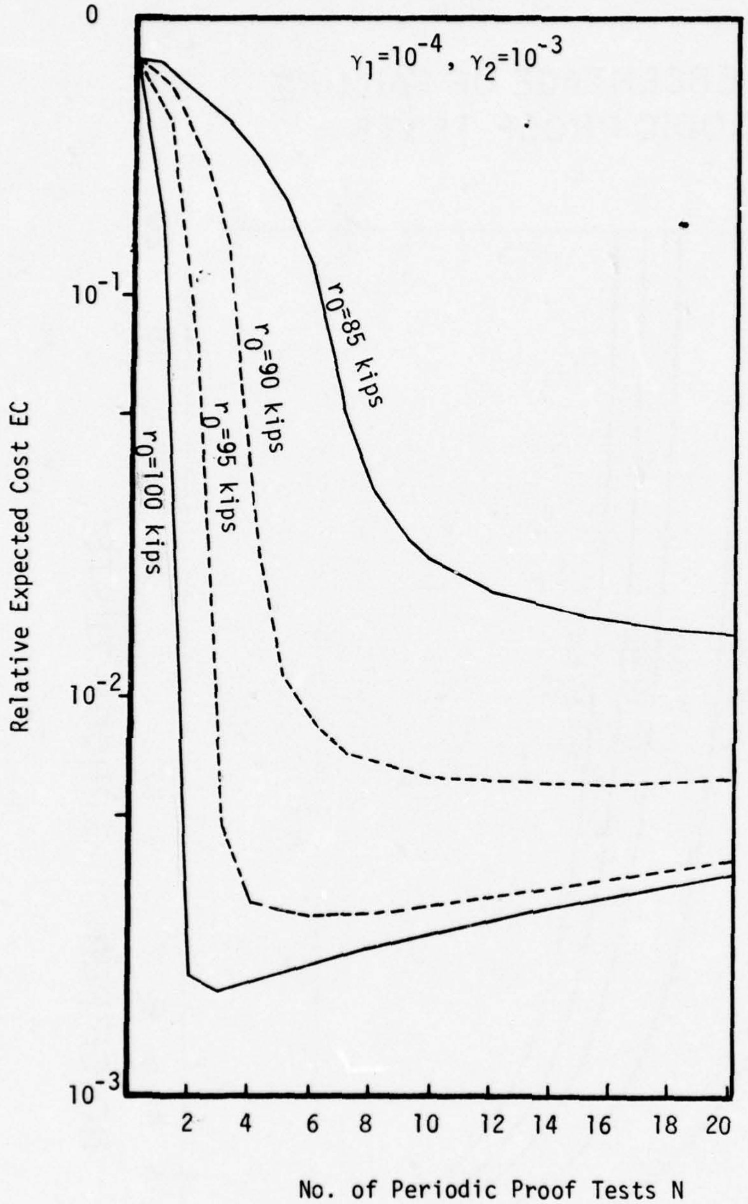

FIG. 12.: RELATIVE EXPECTED COST EC VS NUMBER OF PROOF TESTS FOR VARIOUS VALUES OF γ_1 AND γ_2:
(c) $\gamma_1 = 10^{-4}$, $\gamma_2 = 10^{-3}$

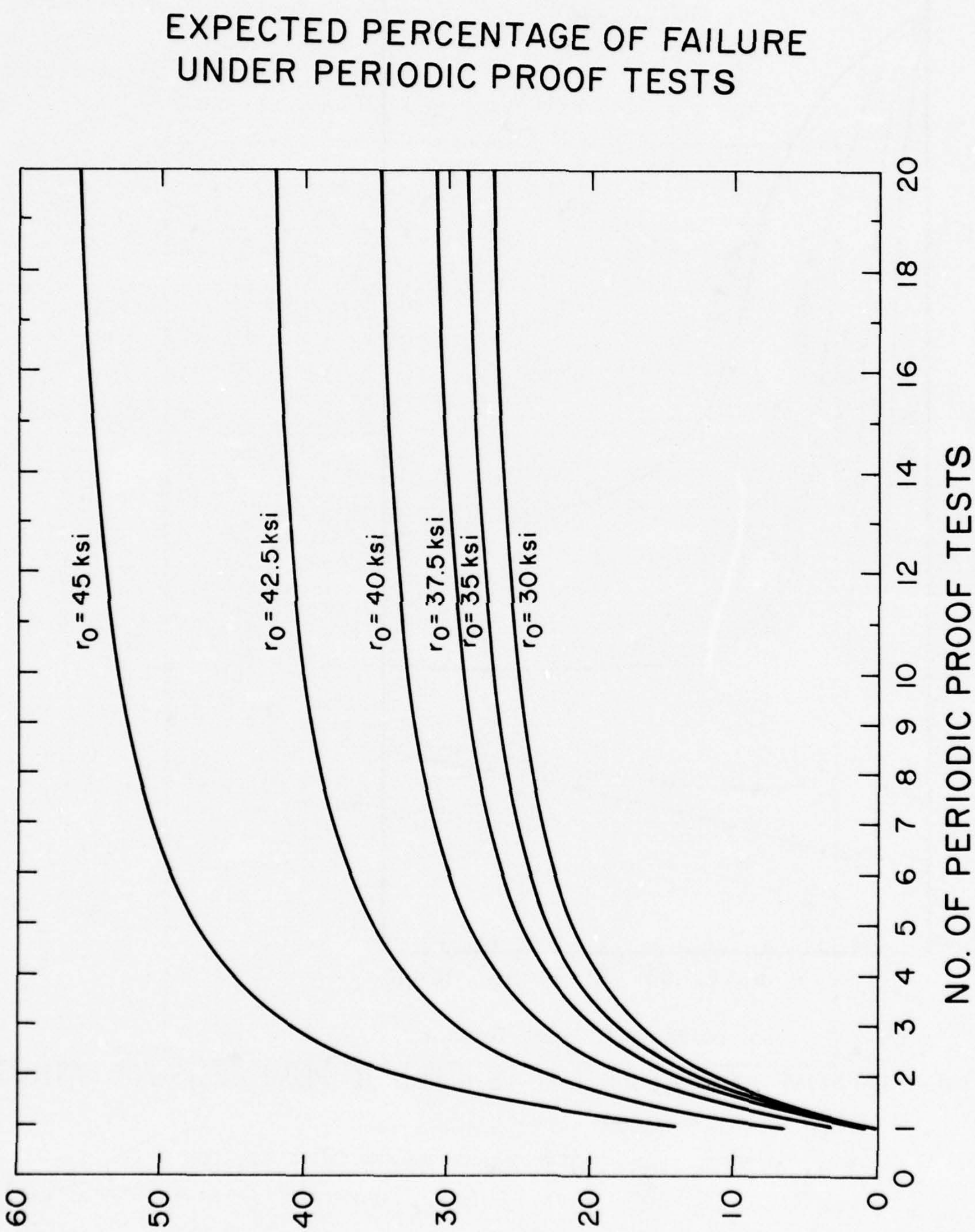

FIG. 13.: EXPECTED PERCENTAGE OF COMPONENTS TO BE DESTROYED BY PERIODIC PROOF TESTS ($\pi/4$ GLASS/EPOXY LAMINATES)

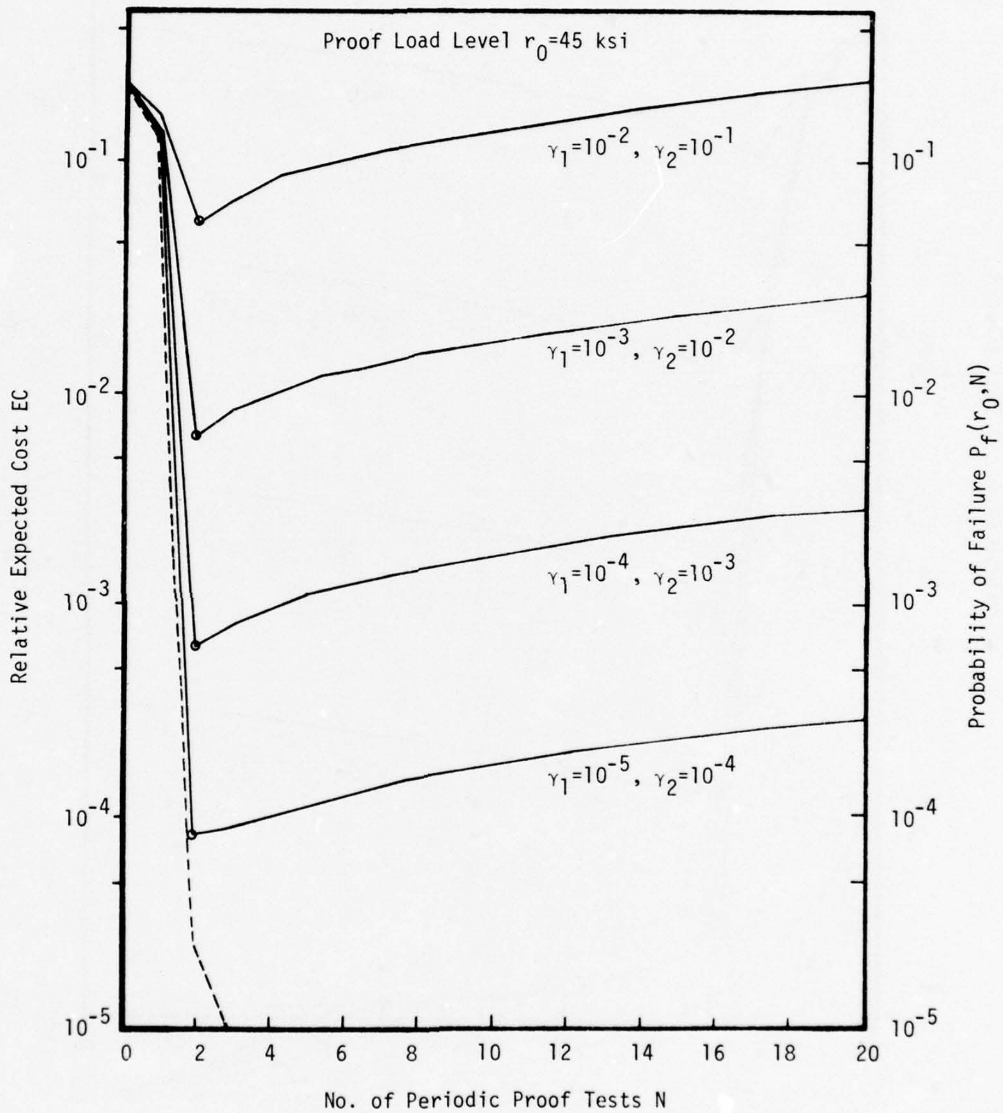

FIG. 14.: RELATIVE EXPECTED COST EC AND PROBABILITY OF FAILURE $P_F(r_0,N)$ VS NUMBER OF PROOF TESTS, N (GLASS/EPOXY $\pi/4$ LAMINATES β = 53 KSI): (A) r_0 = 45 KSI

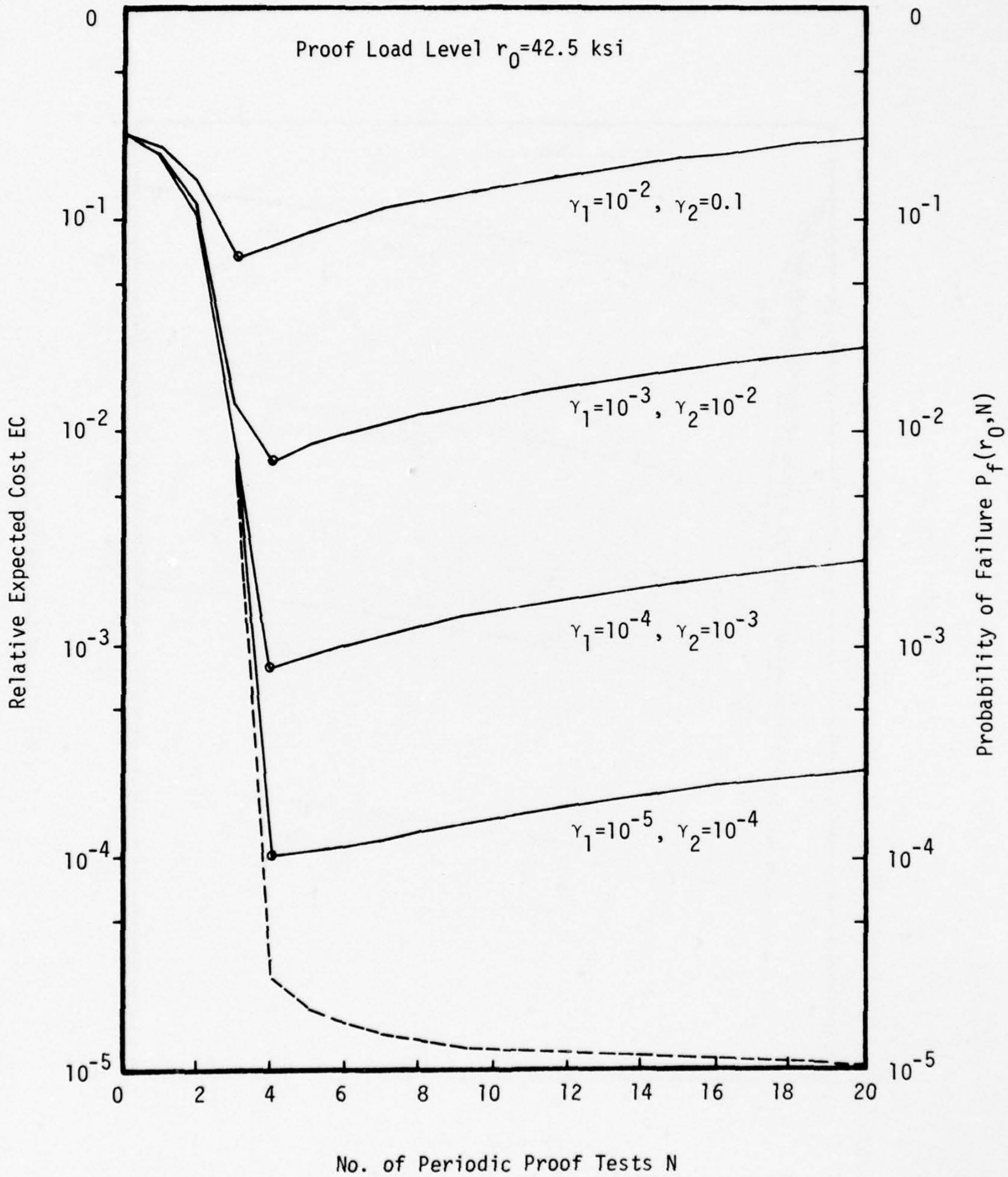

FIG. 14.: RELATIVE EXPECTED COST EC AND PROBABILITY OF FAILURE $P_F(r_0,N)$ VS NUMBER OF PROOF TESTS, N(GLASS/EPOXY $\pi/4$ LAMINATES β = 53 KSI): (B) r_0 = 42.5 KSI

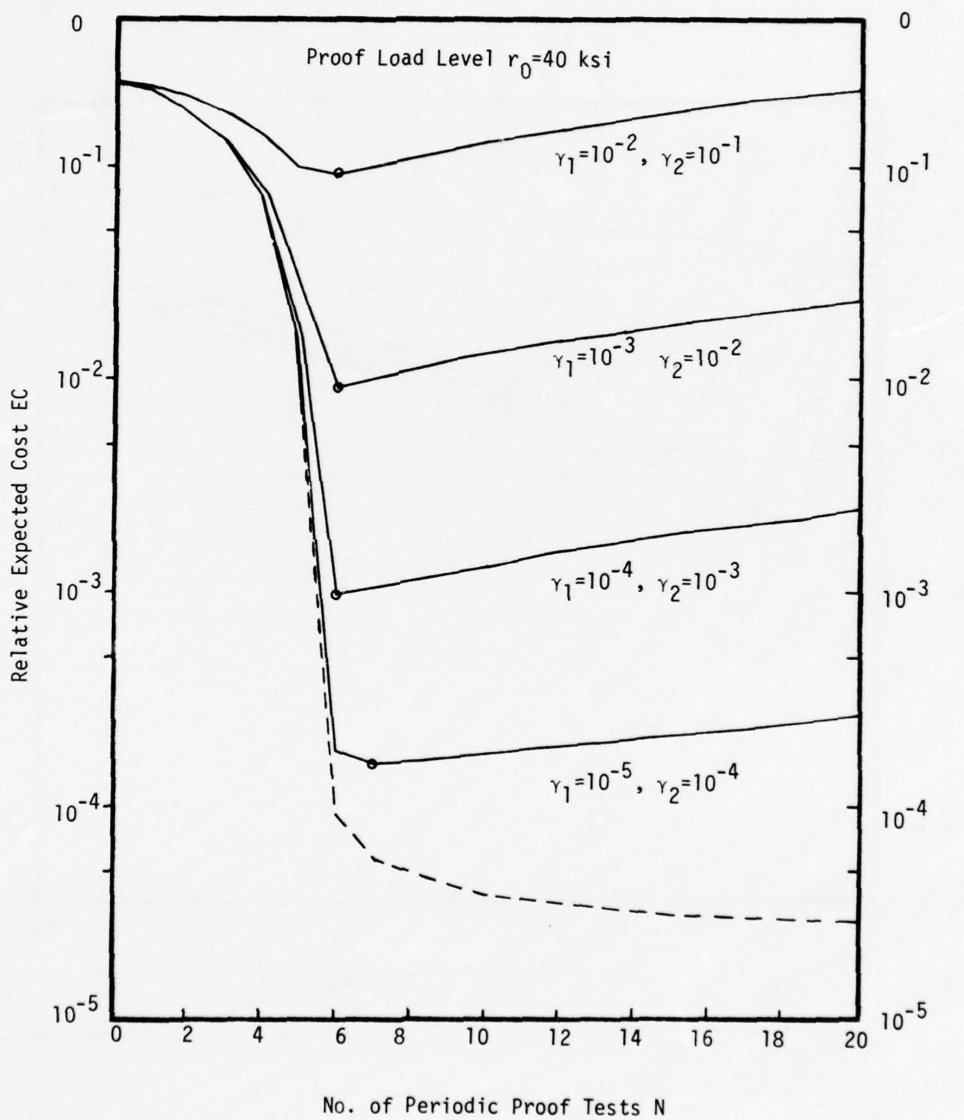

FIG. 14.: RELATIVE EXPECTED COST EC AND PROBABILITY OF FAILURE $P_F(r_0,N)$ VS NUMBER OF PROOF TESTS, N (GLASS/EPOXY $\pi/4$ LAMINATES β = 53 KSI): (c) r_0 = 40 KSI

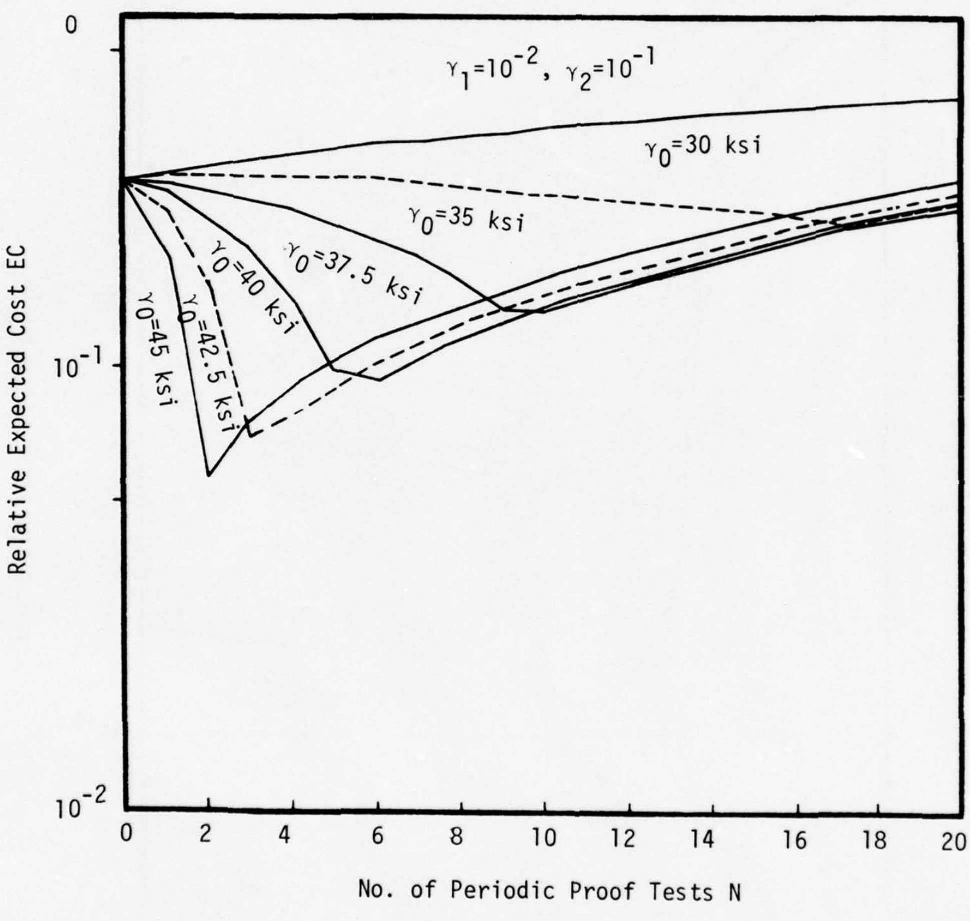

FIG. 15.: RELATIVE EXPECTED COST EC VS NUMBER OF PROOF TESTS FOR VARIOUS VALUES OF γ_1 AND γ_2:

(A) $\gamma_1 = 10^{-2}$, $\gamma_2 = 10^{-1}$

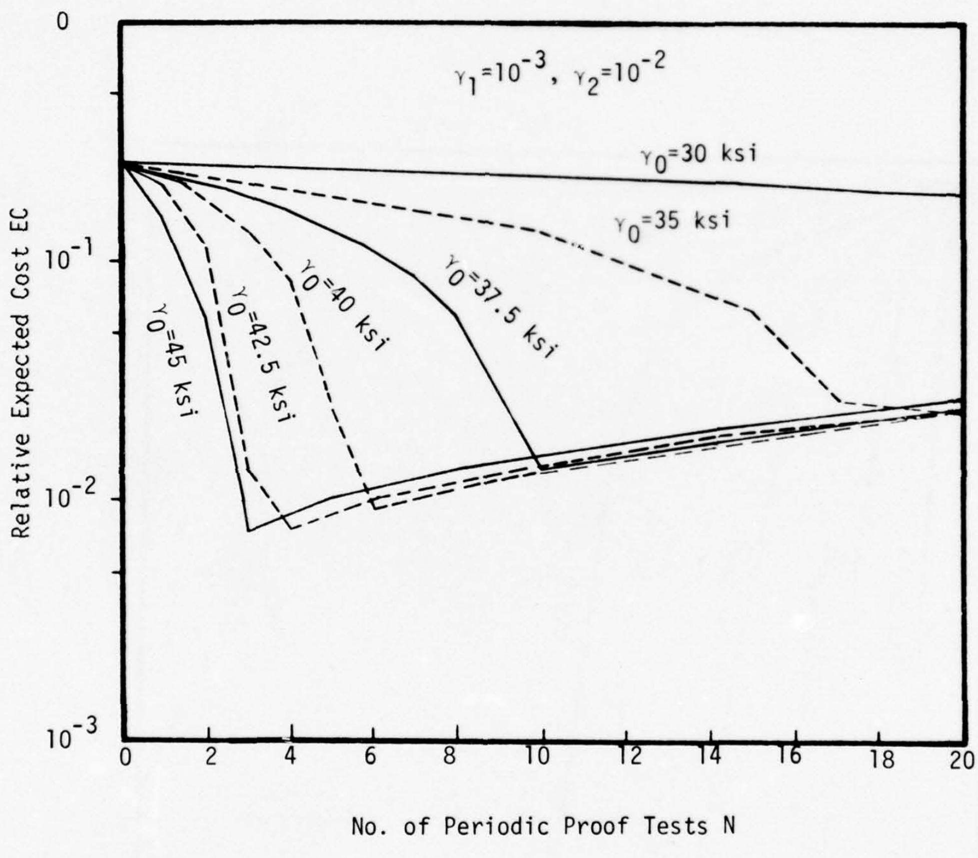

FIG. 15.: RELATIVE EXPECTED COST EC VS NUMBER OF PROOF TESTS FOR VARIOUS VALUES OF γ_1 AND γ_2:
(B) $\gamma_1 = 10^{-3}$, $\gamma_2 = 10^{-2}$

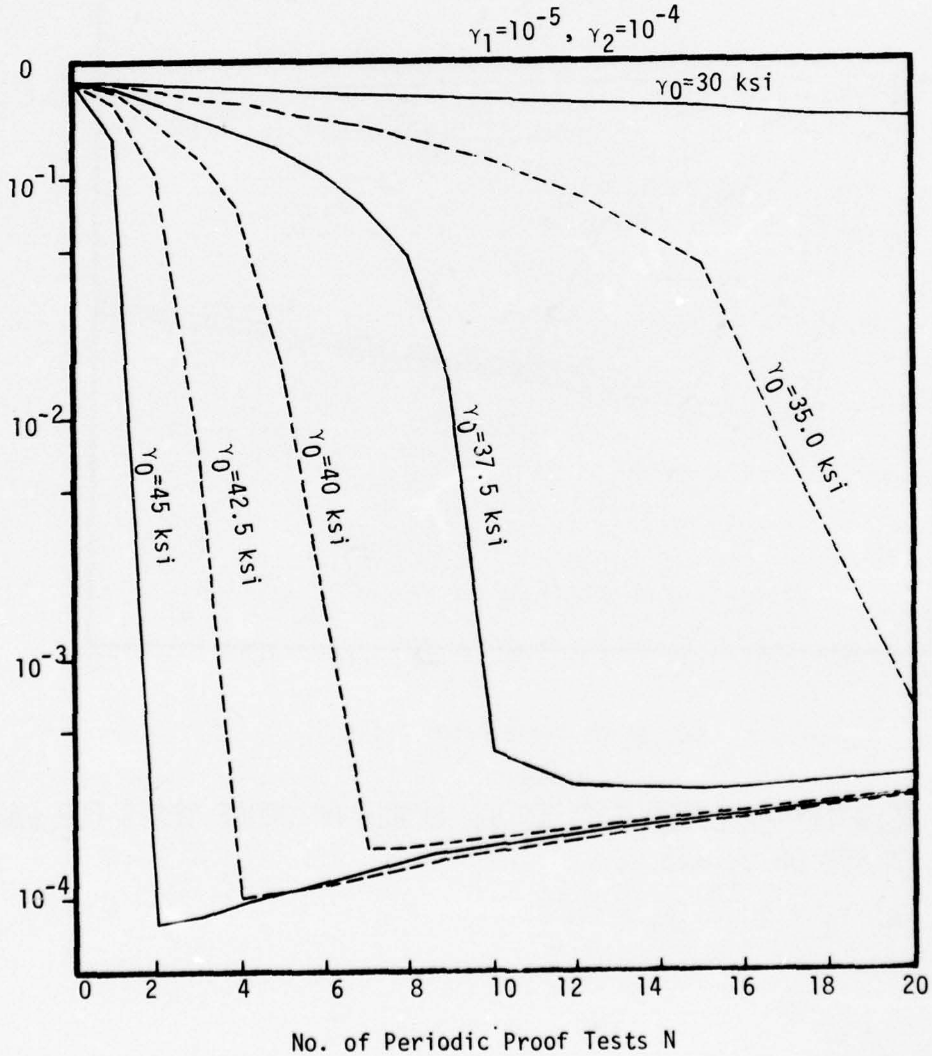

FIG. 15.: RELATIVE EXPECTED COST EC VS NUMBER OF PROOF TESTS FOR VARIOUS VALUES OF γ_1 AND γ_2:
(c) $\gamma_1 = 10^{-5}$, $\gamma_2 = 10^{-4}$

APPENDIX A. STATISTICAL DISTRIBUTION OF RESIDUAL STRENGTH UNDER PERIODIC PROOF TESTS IN SERVICE AND SURVIVAL PROBABILITY (NO DAMAGE DUE TO PROOF TESTS)

The distribution function of the ultimate strength R(0) after the initial (first) proof test prior to service is given by Eq. 2. During service the strength is subjected to degradation following Eq. 4. The residual strength at T flights before the 2nd proof test, denoted by R(T-), is related to R(0) through Eq. 4,

$$R^c\,(T-) = R^c(0) - \Phi T \qquad \text{(A.1)}$$

The distribution function of R(T-) can be obtained from that of R(0) given by Eq. 2 through the transformation of Eq. I-1,

$$F_{R(T-)}(x) = P[R(T-) \leq x] = P[\{R^c(0) - \Phi T\}^{1/c} \leq x]$$

$$= P[R(0) \leq (x^c + \Phi T)^{1/c}] \qquad \text{(A.2)}$$

$$= 1 - \exp\left\{\left(\frac{r_0}{\beta}\right)^{\alpha} - \left(\frac{x^c + \Phi T}{\beta^c}\right)^{\alpha/c}\right\};$$

After the second proof test performed at T flight, the distribution function, $F_{R(T)}(x)$, of the residual strength, R(T), for surviving components can be obtained from $F_{R(T-)}(x)$ as follows;

$$F_{R(T)}(x) = P[R(T) \leq x] = 1 - P[R(T) > x]$$

$$= 1 - P[R(T-) > x \,|\, R(T-) > r_0] \qquad \text{(A.3)}$$

$$= 1 - \frac{P[R(T-) > x]}{P[R(T-) > r_0]} \quad ; \quad x \geq r_0$$

Substitution of Eq. I-2 into Eq. I-3 yields

$$F_{R(T)}(x) = 1 - \exp\left\{\left(\frac{r_0{}^c + \Phi T}{\beta^c}\right)^{\alpha/c} - \left(\frac{x^c + \Phi T}{\beta^c}\right)^{\alpha/c}\right\}; \; x \geq r_0 \tag{A.4}$$

Under the second proof test performed at T, the probability of surviving such a proof test, denoted by B_1^*, is

$$B_1^* = P[R(T-) > r_0] = \exp\left\{\left(\frac{r_0}{\beta}\right)^{\alpha} - \left(\frac{r_0{}^c + \Phi T}{\beta^c}\right)^{\alpha/c}\right\} \tag{A.5}$$

Let R(2T-) be the residual strength at 2T before the 3rd proof test. Then, due to the strength degradation, R(2T-) is related to R(T) through Eq. 4,

$$R^c(2T-) = R^c(T) - \Phi T \tag{A.6}$$

The distribution function of R(2T-) can be obtained from that of R(T), given by Eq. I-4, through the transformation of Eq. I-6 (similar to the transformation given in Eq. I-2) as follows,

$$F_{R(2T-)}(x) = 1 - \exp\left\{\left(\frac{r_0{}^c + \Phi T}{\beta^c}\right)^{\alpha/c} - \left(\frac{x^c + 2\Phi T}{\beta^c}\right)^{\alpha/c}\right\} \tag{A.7}$$

After the third proof test is performed at 2T, the distribution function $F_{T(2T)}(x)$ of the surviving original components can be obtained similarly to the transformation of Eq. I-3,

$$F_{R(2T)}(x) = 1 - \frac{P[R(2T-) > x]}{P[R(2T-) > r_0]} \tag{A.8}$$

Application of Eq. I-7 into Eq. I-8 yields

$$F_{R(2T)}(x) = 1 - \exp\left\{\left[\frac{r_0^{\,c} + 2\Phi T}{\beta^c}\right]^{\alpha/c} - \left[\frac{x^c + 2\Phi T}{\beta^c}\right]^{\alpha/c}\right\};\ x \geq r_0 \tag{A.9}$$

The probability of surviving the 3rd proof test performed at 2T for the original component, denoted by $\tilde{B}_2$, follows from Eq. I-7 as

$$\tilde{B}_2 = P[R(2T-) \geq r_0] = \exp\left\{\left(\frac{r_0^{\,c} + \Phi T}{\beta^c}\right)^{\alpha/c} - \left(\frac{r_0^{\,c} + 2\Phi T}{\beta^c}\right)^{\alpha/c}\right\} \tag{A.10}$$

The probability, B_2^*, of survival up to 2T flights for the original components, i.e., the probability of surviving both the second proof test performed at T and the third proof test performed at 2T, is

$$B_2^* = B_1^* \tilde{B}_2 = \exp\left\{\left(\frac{r_0}{\beta}\right)^{\alpha} - \left(\frac{r_0^{\,c} + 2\Phi T}{\beta^c}\right)^{\alpha/c}\right\} \tag{A.11}$$

in which Eqs. I-5 and I-10 have been used.

In a similar manner, the distribution function of the residual strength right after the j+1th proof test performed at jT, for the original components, can be derived as

$$F_{R(jT)}(x) = 1 - \exp\left\{\left(\frac{r_0^{\,c} + \Phi jT}{\beta^c}\right)^{\alpha/c} - \left(\frac{x^c + \Phi jT}{\beta^c}\right)^{\alpha/c}\right\};\ x \geq r_0 \tag{A.12}$$

and the probability density function, $f_{R(jT)}(x)$, of R(jT) is

$$f_{R(jT)}(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{c-1}\left(\frac{x^c + \Phi jT}{\beta^c}\right)^{\frac{\alpha}{c}-1}$$

$$\exp\left\{\left(\frac{r_0{}^c + \Phi jT}{\beta^c}\right)^{\alpha/c} - \left(\frac{x^c + \Phi jT}{\beta^c}\right)^{\alpha/c}\right\};\ x \geq r_0 \tag{A.13}$$

The probability of surviving all the previous proof tests up to jT flights, denoted by B_j^*, can be shown as

$$B_j^* = \exp\left\{\left(\frac{r_0}{\beta}\right)^{\alpha} - \left(\frac{r_0{}^c + \Phi jT}{\beta^c}\right)^{\alpha/c}\right\} \tag{A.14}$$

APPENDIX B. DAMAGING EFFECT DUE TO PROOF TESTS

Two possible types of damage to composites may occur due to the proof test, referred to as the static strength damage and the fatigue strength damage, respectively, as discussed in the following;

(1) The static strength damage refers to the degradation of the residual strength immediately after proof testing. This resembles the case where a crack exists and the crack size increases under one cycle of large proof load thus resulting in the degradation of strength immediately. Let $R(\tau+)$ be the residual strength right after the proof test performed at τ and $R(\tau-)$ be the residual strength right before the proof test. A simple model is that

$$R(\tau+) = D_1(r_0/\beta)\, R(\tau-) \tag{B.1}$$

in which $D_1(r_0/\beta)$ is a function of r_0/β that is the ratio of the proof load level, r_0, to the characteristic strength, β. Necessary conditions for the damaging function are $D_1(r_0/\beta) = 1$ for $r_0 < r^*$ and $D_1(r_0/\beta) = 0$ for $r_0 \rightarrow \infty$, where r^* is the threshold level under which no damage will occur. A simple and possible form for $D_1(r_0/\beta)$ is

$$\begin{aligned} D_1(r_0/\beta) &= A \exp\left[-a_1(r_0/\beta)^{a_2}\right] \quad ; \quad r_0 \geq r^* \\ &= 1 \quad ; \quad r_0 < r^* \end{aligned} \tag{B.2}$$

in which the parameters a_1 and a_2 have to be determined from test results and A is a constant such that $D_1=1$ for $r_0=r^*$, i.e., $A=\exp[a_1(r^*/\beta)^{a_2}]$.

(2) The fatigue strength damage refers to faster degradation of residual strength after the component being proof tested. This resembles the case where there is no crack exists before the proof test and the crack is introduced during proof testing, e.g., the first ply failure. As a result, the strength degradation accelerates under the subsequent fatigue loads. A simple form of the accelerated strength degradation can be written as

$$R^c(t + \tau) = R^c(\tau+) - \Phi D_2(r_0/\beta)\, t \tag{B.3}$$

which differs from Eq. 4 by a damaging function $D_2(r_0/\beta)$. The necessary conditions for $D_2(r_0/\beta)$ are that $D_2(r_0/\beta) = 1$ for $r_0 < r^*$ and $D_2(r_0/\beta) = \infty$ for $r_0 \rightarrow \infty$.

A simple form for $D_2(r_0/\beta)$ may be

$$\begin{aligned} D_2(r_0/\beta) &= A^* \exp\left[a_3(r_0/\beta)^{a_4}\right] \qquad ; r_0 \geq r^* \\ &= 1 \qquad ; r_0 < r^* \end{aligned} \tag{B.4}$$

in which the parameters a_3 and a_4 have to be determined by test results, and $A^* = \exp[-a_3(r^*/\beta)^{a_4}]$

Accounting for the damaging effects given by Eqs. II-1 and II-3, the probability density function, $f_{R(jT)}(x)$, of the residual strength, $R(jT)$, of the original components right after the j+1th proof test can be shown as follows [see Appendix III]

$$f_{R(jT)}(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{c-1} \frac{1}{D_1^{jc}} \left[\frac{x^c + \Phi D_2 T \sum_{k=1}^{j} D_1^{kc}}{\beta^c D_1^{jc}}\right]^{\frac{\alpha}{c}-1}$$

$$\exp\left\{\left(\frac{r_0^c + \Phi D_2 T \sum_{k=0}^{j-1} D_1^{kc}}{\beta^c D_1^{jc}}\right)^{\alpha/c} - \left(\frac{x^c + \Phi D_2 T \sum_{k=1}^{j} D_1^{kc}}{\beta^c D_1^{jc}}\right)^{\alpha/c}\right\} \tag{B.5}$$

The probability of surviving up to the j+1th proof test performed at jT can be derived as [see Appendix III]

$$B_j^* = \exp\left\{\left(\frac{r_0}{\beta}\right)^{\alpha} - \left(\frac{r_0^c + \Phi D_2 T \sum_{k=0}^{j-1} D_1^{kc}}{\beta^c D_1^{jc}}\right)^{\alpha/c}\right\} \tag{B.6}$$

APPENDIX C. STATISTICAL DISTRIBUTION OF RESIDUAL STRENGTH UNDER PERIODIC PROOF TESTS IN SERVICE AND SURVIVAL PROBABILITY (DAMAGE DUE TO PROOF TESTS)

The distribution function $F_{R_0}(x)$ of the ultimate strength R_0 of composites is given by Eq. 1. During the initial (first) proof testing, not only the tail of the distribution function $F_{R_0}(x)$ is truncated, referred to as the truncation effect, but damage also occurs according to Eqs. II-1 and II-3, referred to as the damaging effect. As a result, the distribution function for the composites, that survive proof testing, is subjected to both the truncation effect, e.g., Eq. I-3, and the damaging effect, Eq. II-1. The distribution function $F_{R(0)}(x)$ of the ultimate strength R(0) for components that survive proof testing without the damaging effect is given by Eq. 2. It follows from Eq. I-2 that the strength R(0) after the initial proof test accounting for the damaging effect is

$$R(0+) = D_1 R(0) \tag{C.1}$$

in which D_1 is written for $D_1(r_0/\beta)$. Hence,

$$\begin{aligned} F_{R(0+)}(x) &= P[R(0+) \leq x] = P[D_1 R(0) \leq x] \\ &= P[R(0) \leq \frac{x}{D_1}] \qquad\qquad (C.2) \\ &= 1 - \exp\left\{\left(\frac{r_0}{\beta}\right)^\alpha - \left(\frac{x}{\beta D_1}\right)^\alpha\right\} \quad ; x \geq D_1 r_0 \end{aligned}$$

in which $F_{R(0+)}(x)$ is the distribution function of strength after the initial proof test where the static strength damaging effect is taken into account.

Due to the strength degradation under service load, the residual strength right before the second proof performed at T flight, R(T-), is related to R(0+) through

$$R^c(T-) = R^c(0+) - \Phi D_2 T \tag{C.3}$$

in which D_2 is written for $D_2(r_0/\beta)$.

The distribution function, $F_{R(T-)}(x)$, of R(T-) can easily be obtained from that of R(0+) given by Eq. III-2 through transformation of Eq. III-3 (see Appendix I) as follows;

$$F_{R(T-)}(x) = 1 - \exp\left\{\left(\frac{r_0}{\beta}\right)^{\alpha} - \left(\frac{x^c + \Phi D_2 T}{\beta^c D_1{}^c}\right)^{\alpha/c}\right\} \tag{C.4}$$

During the second proof testing at T, the truncation effect leads to (see Eq. I-3)

$$F_{R(T)}(x) = 1 - \frac{P[R(T-) > x]}{P[R(T-) > r_0]} = 1 - \exp\left\{\left(\frac{r_0{}^c + \Phi D_2 T}{\beta^c D_1{}^c}\right)^{\alpha/c} - \left(\frac{x^c + \Phi D_2 T}{\beta^c D_1{}^c}\right)^{\alpha/c}\right\} \tag{C.5}$$

and the damaging effect that

$$R(T+) = D_1 R(T) \tag{C.6}$$

leads to the following result,

$$\begin{aligned} F_{R(T+)}(x) &= P[R(T+) \leq x] = P[R(T) \leq \frac{x}{D_1}] \\ &= 1 - \exp\left\{\left(\frac{r_0{}^c + \Phi D_2 T}{\beta^c D_1{}^c}\right)^{\alpha/c} - \left(\frac{x^c + \Phi D_2 T D_1^c}{\beta^c D_1^{2c}}\right)^{\alpha/c}\right\} \end{aligned} \tag{C.7}$$

in which Eq. III-5 has been used.

The probability of surviving the proof test performed at T flights, denoted by B_1^* is

$$B_1^* = P[R(T-) > r_0] = \exp\left\{\left(\frac{r_0}{\beta}\right)^{\alpha} - \left(\frac{r_0^{\,c} + \Phi D_2 T}{\beta^c D_1^{\,c}}\right)^{\alpha/c}\right\} \tag{C.8}$$

The degradation effect in the 2nd service interval is (see Eq. II-3)

$$R^c(2T-) = R^c(T+) - \Phi D_2 T \tag{C.9}$$

Hence, the distribution function, $F_{R(2T-)}(x)$, of the residual strength, $R(2T-)$, right before the 3rd proof test performed at 2T can be obtained from Eq. III-7 through the transformation of Eq. III-9,

$$\begin{aligned} F_{R(2T-)}(x) &= P[R(2T-) \leq x] = P[R(T+) \leq (x^c + \Phi D_2 T)^{1/c}] \\ &= 1 - \exp\left\{\left(\frac{r_0^{\,c} + \Phi D_2 T}{\beta^c D_1^{\,c}}\right)^{\alpha/c} - \left(\frac{x^c + \Phi D_2 T + \Phi D_2 T D_1^{\,c}}{\beta^c D_1^{2c}}\right)^{\alpha/c}\right\} \end{aligned} \tag{C.10}$$

During the 3rd proof test performed at 2T, the truncation effect results in (see Eq. I-8),

$$\begin{aligned} F_{R(2T)}(x) &= 1 - \frac{P[R(2T-) > x]}{P[R(2T-) > r_0]} \\ &= 1 - \exp\left\{\left(\frac{r_0^{\,c} + \Phi D_2 T + \Phi D_2 T D_1^{\,c}}{\beta^c D_1^{2c}}\right)^{\alpha/c} - \left(\frac{x^c + \Phi D_2 T + \Phi D_2 T D_1^{\,c}}{\beta^c D_1^{2c}}\right)^{\alpha/c}\right\} \end{aligned} \tag{C.11}$$

and the damaging effect $R(2T+) = D_1 R(2T)$ leads to

$$F_{R(2T+)}(x) = P\left[R(2T) \le \frac{x}{D_1}\right]$$

$$= 1 - \exp\left\{\left(\frac{r_0^c + \Phi D_2 T + \Phi D_2 T D_1{}^c}{\beta^c D_1^{2c}}\right)^{\alpha/c} - \left(\frac{x^c + \Phi D_2 T D_1^c + \Phi D_2 T D_1^{2c}}{\beta^c D_1^{3c}}\right)^{\alpha/c}\right\}$$

(C.12)

The probability of surviving only the third proof test performed at 2T, denoted by $\tilde{B}_2$, is

$$\tilde{B} = P[R(2T-) > r_0] = \exp\left\{\left(\frac{r_0{}^c + \Phi D_2 T}{\beta^c D_1{}^c}\right)^{\alpha/c} - \left(\frac{r_0{}^c + \Phi D_2 T + \Phi D_2 T D_1{}^c}{\beta^c D_1^{2c}}\right)^{\alpha/c}\right\}$$

(C.13)

Therefore, the probability of surviving both the second and the third proof test for the original component, denoted by B_2^*, is

$$B_2^* = B_1^* \tilde{B}_2 = \exp\left\{\left(\frac{r_0}{\beta}\right)^{\alpha} - \left(\frac{r_0{}^c + \Phi D_2 T + \Phi D_2 T D_1{}^c}{\beta^c D_1^{2c}}\right)^{\alpha/c}\right\} \quad \text{(C.14)}$$

In a similar manner, the distribution function, $F_{R(jT+)}(x)$, of the residual strength, $R(jT+)$, right after the j+1th proof test performed at jT can be derived for the original component as follows;

$$F_{R(jT+)}(x) = 1 - \exp\left\{\left(\frac{r_0{}^c + \Phi D_2 T \sum_{k=0}^{j=1} D_1{}^{kc}}{\beta^c D_1{}^{jc}}\right)^{\alpha/c} - \left(\frac{x^c + \Phi D_2 T \sum_{k=1}^{j} D_1{}^{kc}}{\beta^c D_1{}^{(j+1)c}}\right)^{\alpha/c}\right\}$$

(C.15)

The probability of surviving all proof tests (except the initial one) up to jT for the original component can similarily be derived as follows;

$$B_j^* = \exp \left\{ \left(\frac{r_0}{\beta}\right)^{\alpha} - \left(\frac{r_0^{\,c} + \Phi D_2 T \sum_{k=0}^{j-1} D_1^{\,kc}}{\beta^c D_1^{\,jc}} \right)^{\alpha/c} \right\} \qquad \text{(C.16)}$$

It can easily be observed that Eqs. III-15 and III-16 degenerate into Eqs. 5 and 7 when no damage is inflicted by proof tests, in which case $D_1 = 1$, $D_2 = 1$.

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